The University of the State of New York

EXAMINATION FOR QUALIFYING CERTIFICATES

PLANE TRIGONOMETRY

Wednesday, September 14, 1927—9.15 a. m. to 12.15 p. m., only

Answer seven questions, including three from group I and four from group II. Papers entitled to less than 75 credits will not be accepted.
A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.
Give special attention to neatness and arrangement of work.

Group I

Answer three questions from this group.

1 Solve the right triangle in which $c = 1843.7$ and $b = 618.42$. [16]

2 Given $a = 77.8$, $b = 114.3$, $c = 182.2$; find C. [16]

3 To find the distance from a point A to another point B, the latter being inaccessible and invisible from A, two points C and D are selected so that C, A and D are in one straight line and so that A and B are visible both from C and from D. By measurement it is found that $CA = 456.7$ feet, $AD = 490.7$ feet, angle $C = 71° 23'$, angle $D = 36° 19'$; find the distance AB. [16]

4 At a point A the angle of elevation DAB to the top of a vertical wall is $m°$ and the angle of depression CAD to its base is $n°$. If the horizontal distance AD is equal to a feet, prove that CB, the height of the wall, may be expressed in *each* of the following forms:

$a(\tan m° + \tan n°)$; $\dfrac{a \sin(m° + n°)}{\cos m° \cos n°}$ [12, 4]

Group II

Answer four questions from this group.

5 *a* Starting with the formula for $\cos 2x$, derive the formula

$$\tan x = \frac{\sin 2x}{1 + \cos 2x}$$ [10]

b Express $\sin x$ in terms of $\tan x$. [3]

6 Prove the following identity:

$$\sin(45° - \tfrac{1}{2}x) + \cos(45° - \tfrac{1}{2}x) = \frac{\sin x}{\sqrt{1 - \cos x}}$$ [13]

7 The angle of elevation of a tower QP (Q being the base) from a point A due south is $r°$ and from a point B due west of the first station is $s°$. If the distance AB between the two stations is b, show that the height of the tower is

$$\frac{b \sin r° \sin s°}{\sqrt{\sin(r° + s°) \sin(r° - s°)}}$$

[Note: Angles PQA, PQB, PAB and QAB are right angles.] [13]

8 *a* Prove without the use of tables

$$\sec 105° = -\sqrt{2}(\sqrt{3} + 1)$$ [8]

b If $x = \sin^{-1}(-\frac{3}{5})$ and is an angle in the third quadrant, find $\tan x$. [5]

9 Solve the following equation for all positive values of x between 0° and 360°: $2 \sin^2 x + \sin^2 2x = 2$ [13]

10 *a* Show by a drawing that $\sin(90° + x) = \cos x$, when x is an angle less than 90°. [4]

b In a circle whose radius is unity, show how to represent as a line *each* of the following functions of x when x is an angle less than 90°: $\sin x$, $\cos x$, $\tan x$. [9]

11 State whether *each* of the following statements is true or false: [Label each answer with the corresponding letter.]

a Cos $(90° + x)$ is equal to the negative of the cosine of the supplementary angle. [3]

b Tan A can never equal $-\cot A$. [2]

c The logarithm of the tangent of an angle increased by the logarithm of the tangent of the complement of the same angle always equals zero. [3]

d The equation $\sin 2A = 2 \sin A$ is true for all values of A. [2]

e Sin $(x - y) = -\sin(x + y)$ [3]