The University of the State of New York

EXAMINATION FOR QUALIFYING CERTIFICATES

PLANE TRIGONOMETRY

Wednesday, September 13, 1922—9.15 a. m to 12.15 p. m., only

Answer seven questions, including two from group I, two from group II and three from group III. Papers entitled to less than 75 credits will not be accepted.

A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

Group I

Answer two questions from this group.

1 Assuming the formulas for $\sin (A + B)$ and $\cos (A + B)$, derive the following formulas:

$$\sin 2A;\ \cos 2A;\ \sin \frac{A}{2};\ \cos \frac{A}{2} \qquad [13]$$

2 Prove that in any triangle the square on any side is equal to the sum of the squares on the other two sides diminished by twice their product into the cosine of their included angle. [13]

3 Prove that in any triangle $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ where a, b and c are the sides of the triangle, s is one half the perimeter and A is the angle opposite side a. [13]

Group II

Answer two questions from this group.

4 *a* Prove that $(\tan A + \sec A)^2 = \frac{1 + \sin A}{1 - \sin A}$ [7]

b Prove that $\frac{\cot B - \tan B}{\cot B + \tan B} = \cos 2B$ [6]

5 Solve the following equation for values of x between 0° and 360°:

$$\frac{1 - \tan x}{1 + \tan x} = \cos 2x \qquad [13]$$

6 *a* If $\sin A = \frac{4}{5}$ and $\cos A = -\frac{12}{13}$ find the value of $\tan 2A$. In what quadrant is $2A$? [7, 2]

b Express in degrees $1\frac{1}{4}\pi$ radians.
Express in radians 120° 45' [4]

142

Group III

Answer three questions from this group.

7 A building 52.6 feet high stands at the foot of a hill; from the top of the hill the angles of depression of the top and the bottom of the building are 45° 19' and 47° 52' respectively. What is the height of the hill? [16]

8 The sides of a triangle are 78.945, 65.48, 97.315. Find the largest angle. [16]

9 By the use of logarithms find the value of

$$\left(\frac{\sqrt[5]{78.16 \times .06271}}{6.1257 \times .1321}\right)^3 \qquad [16]$$

10 Wishing to determine the distance between a building A and a tower B, on opposite sides of a river, a man measured a line CD along the river (C being nearly opposite A) and observed the following angles:

$\angle ACB = 58° 20'$; $\angle ACD = 95° 20'$; $\angle ADB = 53° 30'$; $\angle BDC = 98° 45'$. CD is 600'. What is the required distance? [Outline in detail the work necessary for the solution and the order for doing the work, but *do not do any of the logarithmic computation.*] [16]