

The University of the State of New York

285TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Friday, August 21, 1942 — 3.30 to 6.30 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Part II

Write at top of first page of answer paper to part II (a) names of schools where you have studied, (b) number of weeks and recitations a week in plane trigonometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1942.

The minimum time requirement is five recitations a week for half a school year, or the equivalent. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1942 is required.

In this examination the customary lettering is used. A , B and C represent the angles of a triangle ABC ; a , b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

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See instructions for part II on page 1.

Part II

Answer question 21 and four of the others.

21 In triangle ABC , $a = 32.42$, $b = 30.00$ and $c = 25.58$. Find C correct to the *nearest minute* by using the following outline:

$$s = \frac{1}{2} (a + b + c) = 44.00$$

$$\sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$\log (s-a) = \dots\dots\dots [1]$$

$$\log \frac{(s-a)(s-b)}{ab} = \dots\dots\dots [1]$$

$$\log (s-b) = \dots\dots\dots [1]$$

$$\log (s-a)(s-b) = \dots\dots\dots [1]$$

$$\log \sin \frac{C}{2} = \dots\dots\dots [1]$$

$$\log a = \dots\dots\dots [1]$$

$$\frac{C}{2} = \dots\dots\dots [1]$$

$$\log b = \dots\dots\dots [1]$$

$$\log ab = \dots\dots\dots [1]$$

$$C = \dots\dots\dots [1]$$

22 *a* On one set of axes draw the graphs of $y = \sin 3x$ and $y = \sin x$, from $x = 0^\circ$ to $x = 180^\circ$ inclusive in intervals of 30° . [5, 3]

b From the graphs, find the value of x between 90° and 180° that makes $\sin 3x$ equal to $\sin x$. [2]

23 *a* Find the values of x between 0° and 360° that satisfy the following equation:

$$4 \sin^2 x - 4 \cos x = 1 \quad [4]$$

b Find the values of A between 0° and 360° that satisfy the following equation:

$$\frac{4}{\cos^2 A} - 7 \tan^2 A - 3 = 0 \quad [6]$$

24 Derive the formula for $\sin (x + y)$ in which x , y and $x + y$ are positive acute angles. [10]

25 The sound of a cannon fired at A is heard at B one second after it is fired and at C two seconds after it is fired. If angle BAC is 75° , find the distance from B to C . [The sound traveled at the rate of 1110 feet per second.] [10]

26 From the top of a hill, the angles of depression of the top and bottom of a 30-foot vertical pole are 44° and 47° respectively. The pole and the hill stand on level ground. How high is the hill? [6, 4]

27 As a boat sails north, two lighthouses, known to be 10 miles apart, are observed to be in line with the boat and directly east of it. Half an hour later their bearings as observed from the boat are $S. 57^\circ E.$ and $S. 71^\circ E.$ Find in miles per hour the rate at which the boat is sailing. [6, 4]

*28 *a* Make a table of values for $r = \sin 2\theta$ from $\theta = 0^\circ$ to $\theta = 180^\circ$ in intervals of 15° . [3]

b Using the values found in answer to *a*, draw the graph of $r = \sin 2\theta$. [r and θ represent the polar coordinates of a variable point.] [7]

* This question is based on one of the optional topics in the syllabus.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 Round off the number 0.024676 to *three* significant figures. 1.....
- 2 Express $\frac{\pi}{4}$ radians in degrees. 2.....
- 3 If $\frac{1}{2}(A + B) = 25^\circ 2'$ and $\frac{1}{2}(A - B) = 10^\circ 13'$, find B . 3.....
- 4 As angle A increases from 180° to 270° , what function of A decreases from 0 to -1 ? 4.....
- 5 Find the area of triangle ABC if $a = 2$, $b = 8$ and $C = 60^\circ$. [Answer may be left in radical form.] 5.....
- 6 Is $\cos x$ (a) always equal to $\cos(-x)$, (b) sometimes equal to $\cos(-x)$ or (c) never equal to $\cos(-x)$? [Answer a, b or c.] 6.....
- 7 How many different triangles can be formed in which $a = 7$, $b = 14$ and $A = 30^\circ$? 7.....
- 8 If $x = 60^\circ$, what is the value of $2 \cos \frac{1}{2} x$? 8.....
- 9 Find the value of $\log \sin 24^\circ 42'$. 9.....
- 10 Solve the equation $1 + \cot x = 0$ for all values of x between 0° and 360° . 10.....
- 11 The angle of depression of a submarine viewed from a point on the top of a perpendicular cliff 400 feet high is 58° . Find, correct to the *nearest foot*, the distance of the submarine from the foot of the cliff. 11.....
- 12 A ship is sailing N. 52° W. at the rate of 20 miles per hour. How many miles to the west will it have sailed in $2\frac{1}{2}$ hours? 12.....
- 13 Express $\sin(180^\circ + A)$ as a function of A . 13.....
- 14 Find the value of $\cos 212^\circ$. 14.....
- 15 Express $\sin(-122^\circ)$ as a function of a positive acute angle. 15.....
- 16 Through how many radians does a spoke of a wheel pass in one revolution? 16.....
- 17 If the cosine of an angle of a triangle is $\frac{4}{5}$, what is the sine of an angle twice as large? 17.....
- 18 If the cosine of an angle of a triangle is $-\frac{1}{2}$, what is the cosine of an angle half as large? 18.....
- 19 Find the positive value of $\tan(\sin^{-1} \frac{3}{5})$. 19.....
- 20 Express $\sin 4x + \sin 2x$ as a product. 20.....