The University of the State of New York

282D HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Friday, August 22, 1941 — 3.30 to 6.30 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Part II

Write at top of first page of answer paper to part II (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1941.

The minimum time requirement is five recitations a week for half a school year, or the equivalent. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1941 is required.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

See instructions for part II on page 1.

Part II

Answer question 21 and four of the others.

21 In triangle ABC, $a = 42.81$, $b = 35.29$, $c = 30.56$. Find C by using the following outline:

$s = \frac{1}{2}(a + b + c) = 54.33$

$\log s = \ldots\ldots\ldots\ldots\ldots\ldots [1]$

$\log (s - c) = \ldots\ldots\ldots\ldots\ldots\ldots [1]$

$\log s(s - c) = \ldots\ldots\ldots\ldots\ldots\ldots [1]$

$\log a = \ldots\ldots\ldots\ldots\ldots\ldots [1]$

$\log b = \ldots\ldots\ldots\ldots\ldots\ldots [1]$

$\log ab = \ldots\ldots\ldots\ldots\ldots\ldots [1]$

$\cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

$\log \frac{s(s-c)}{ab} = \ldots\ldots\ldots\ldots\ldots\ldots [1]$

$\log \cos \frac{C}{2} = \ldots\ldots\ldots\ldots\ldots\ldots [1]$

$\frac{C}{2} = \ldots\ldots\ldots\ldots\ldots\ldots [1]$

$C = \ldots\ldots\ldots\ldots\ldots\ldots [1]$

22 If x is a positive acute angle, find, correct to the *nearest minute,* the values of x that satisfy the equation $6 \sin^2 x + 5 \cos x - 7 = 0$. [10]

23 *a* Prove that the area of triangle $ABC = \frac{1}{2} ab \sin C$ for the case where C is obtuse. [5]

b Show that $\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A}$ is equivalent to $2 \csc A$. [5]

24 A man standing at A observes that the angle of elevation of the top of a hill is 18° 20′. He walks along a horizontal path until he is at B, 1000 feet nearer the hill, and then notes that the angle of elevation is 30° 50′. How far is B from the top of the hill? [10]

25 *a* Draw the graph of $y = 2 \cos \frac{1}{2} x$ from $x = 0°$ to $x = 360°$ inclusive in intervals of 60°. [8]

b Draw the ordinate that represents $\cos 255°$. [2]

26 From the top of a cliff k feet above sea level the angles of depression of two boats were observed to be A and B $(A > B)$. If the observations were made at a moment when the boats and the observer's transit were in the same vertical plane, prove that the distance between the boats was $\frac{k \sin (A - B)}{\sin A \sin B}$ [10]

27 In triangle ABC, $A = 104°$, $b = 251$ and $c = 402$. Find B. [10]

*28 *a* Prove that the equations $x^2 + y^2 = 2x$ and $r = 2 \cos \theta$ represent the same curve. [8]

b What is the name of this curve? [2]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 If $\tan x = k$, express $\tan 2x$ in terms of k. 1.....

2 In triangle ABC, $a = 2$, $b = 5$ and $c = 6$. Express $\cos A$ as a decimal fraction. 2.....

3 As an angle increases from 0° to 360°, what is the maximum value of the logarithm of its sine? 3.....

4 Find the value of A between 0° and 45° which satisfies the equation $\sin 3A = \cos 3A$. 4.....

5 Express $\cos x\ (1 + \tan^2 x)$ in terms of $\cos x$. 5.....

6 Express $\tan\left(x + \frac{\pi}{4}\right)$ as a function of $\tan x$. 6.....

7 Find the numerical value of $\tan(-350°)$. 7.....

8 If $x = \sin^{-1}\frac{\sqrt{3}}{2}$ and x is a positive obtuse angle, how many degrees does it contain? 8.....

9 If A is a positive acute angle whose cosecant is $\sqrt{17}$, what is its cotangent? 9.....

10 Find $\cos\frac{A}{2}$ when A is a positive acute angle whose cosine is $\frac{7}{25}$. 10.....

11 Find $\tan 73° 16'$. 11.....

12 Find, correct to the *nearest minute*, the angle of inclination of a road which rises 1 foot in every 7 measured along the road. 12.....

13 Without using tables, find the numerical value of $2 \sin 15° \cos 15°$. 13.....

14 How many significant figures are there in the number 430.0? 14.....

15 A revolving beacon light strikes a point A, 400 feet east of the beacon, at 11 p. m. and a point B, 400 feet directly north of A, 3 seconds later. How many seconds after 11 o'clock will it strike B the second time? 15.....

16 A block of glass weighing 100 pounds is held on a smooth plane inclined at an angle of 40° by means of a rope parallel to the plane. Find in pounds the pull on the rope. 16.....

17 In triangle ABC, $A = 75°$ and $c = 10$. Express, correct to the *nearest hundredth*, the smallest value that a can have. 17.....

Directions (questions 18–20) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a, b, c or d.

18 An angle whose tangent is negative and whose sine is positive lies (a) in the first quadrant, (b) in the second quadrant, (c) in the third quadrant or (d) in the fourth quadrant. 18.....

19 As an angle increases from 90° to 180°, its cosine (a) increases from 0 to 1, (b) decreases from 1 to 0, (c) increases from —1 to 0 or (d) decreases from 0 to —1. 19.....

20 As an angle increases from 0° to 45°, the function which changes the least is (a) sine, (b) cosine, (c) tangent or (d) cotangent. 20.....