The University of the State of New York

279TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, August 22, 1940 — 3.30 to 6.30 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1940.

The minimum time requirement is five recitations a week for half a school year, or the equivalent. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1940 is required.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Fill in the following lines:

Name of school..... Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Complete the formula $\cos x \cos y - \sin x \sin y = \ldots$ 1.....

2 What is the number whose logarithm is 3.6983? 2.....

3 If $\log \tan \frac{A}{2} = 9.1385\text{–}10$, what is the value of A? 3.....

4 Express the secant of a positive acute angle X in terms of its tangent. 4.....

5 Find log cot 27° 12′. 5.....

6 What is the value of $\sin (-240°)$? 6.....

7 Express $\sin^2 \frac{1}{2}A$ in terms of $\cos A$. 7.....

8 Express $\cos A$ in terms of $\cos 2A$, if A is a positive acute angle. 8.....

9 In triangle ABC, $a = 9$, $b = 7$ and $c = 5$; find the value of $\cos A$. 9.....

10 If $A = 56°$, $b = 10$ feet and $c = 20$ feet, find, correct to the *nearest square foot*, the area of triangle ABC. 10.....

11 In triangle ABC, if $A = 30°$, $B = 105°$ and $a = 20$, what is the value of c? [Answer may be left in radical form.] 11.....

12 What value of x between 90° and 180° satisfies the equation $2 \sin^2 x + 3 \sin x = 2$? 12.....

13 If x is a positive acute angle, what trigonometric function of x can increase from $\frac{1}{2}$ to 2 as x increases? 13.....

14 Express in terms of π the number of degrees in a central angle which intercepts an arc twice as long as the radius of the circle. 14.....

15 If forces of 15 pounds and 8 pounds act on a body at right angles to each other, find, correct to the *nearest degree*, the angle which the resultant will make with the 15-pound force. 15.....

Directions (questions 16–20) — Indicate the correct answer to each question by writing on the line at the right the letter *a*, *b*, *c* or *d*.

16 $\text{Cos}^{-1} \frac{5}{6}$ is equal to (*a*) 33° 33′, (*b*) 33° 34′, (*c*) 33° 35′ or (*d*) 33° 36′. 16.....

17 Log $\frac{a^2}{b}$ is equal to (*a*) $2 \log a - b$, (*b*) $\log 2a - \log b$, (*c*) $2a - b$ or (*d*) $2 \log a - \log b$. 17.....

18 As A increases from 180° to 270 °, $\sin A$ (*a*) increases from 0 to 1, (*b*) decreases from 1 to 0, (*c*) decreases from 0 to —1, or (*d*) increases from — 1 to 0. 18.....

19 The number of values of x which satisfy the equation $2 \csc^2 x + \csc x = 0$ is (*a*) one, (*b*) two, (*c*) none or (*d*) an infinite number. 19.....

20 As x increases from 0° to 360° inclusive, the number of points in which the graph of $\sin x$ intersects the graph of $\sin \frac{1}{2} x$ is (*a*) one, (*b*) two, (*c*) three or (*b*) four. 20.....

See instructions for group II on page 1.

Group II

Answer question 21 and four of the others.

21 In triangle ABC, $A = 86° 18'$, $b = 39.82$ and $c = 15.32$; find C by copying and completing the following outline:

$$\tan \tfrac{1}{2}(B - C) = \frac{(b - c)\tan \tfrac{1}{2}(B + C)}{b + c}$$

$b - c$ = ...[1]

$\tfrac{1}{2}(B + C)$ = ...[1]

$b + c$ = ...[1]

$\log (b - c)$ = ...[1]

$\log \tan \tfrac{1}{2}(B + C)$ = ...[1]

$\log [(b - c)\tan \tfrac{1}{2}(B + C)]$ = ...[1]

$\log (b + c)$ = ...[1]

$\log \tan \tfrac{1}{2}(B - C)$ = ...[1]

$\tfrac{1}{2}(B - C)$ = ...[1]

C = ...[1]

22 *a* Solve for positive values of x less than $360°$: $2 + \cos 2x = 2 \sin^2 x$ [6]

b Express $(\sin x - \cos x)^2$ in terms of $\sin 2x$. [4]

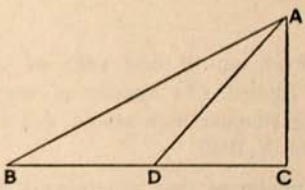
23 *a* Starting with the formula for the tangent of the sum of two angles, derive the formula for the tangent of the double angle. [4]

b Starting with the formulas for the sine of the sum and the sine of the difference of two angles, derive the formula: $\sin x + \sin y = 2 \sin \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y)$ [6]

24 The height of an object, AC, in the figure at the right, may be found by using the formula:

$$AC = \frac{BD}{\cot DBA - \cot CDA}$$

Derive this formula. [10]



25 A straight line, AB, 200 feet long, is measured along one bank of a river. C is an object on the opposite bank. The angles BAC and CBA are observed to be $67° 40'$ and $43° 30'$ respectively. Find the width of the river at C. [10]

26 In triangle ABC, $a = 5.43$, $b = 4.81$ and $c = 3.02$. Using logarithms, find A correct to the *nearest minute*. [10]

27 From a certain point, the angle of elevation of a balloon 5000 feet high was observed to be $35° 20'$. Ten minutes later, from the same point, the elevation was $47° 50'$. If the balloon ascended uniformly and vertically, how fast did it move? [10]

*28 *a* Express $\dfrac{2 + 3i}{5 + 4i}$ in the form $a + bi$. [3]

b Express $6 + 6i$ in its trigonometric form. [4]

c Express $3(\cos 210° + i \sin 210°)$ in the form $a + bi$. [3]

* This question is based on one of the optional topics in the syllabus.

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