The University of the State of New York

276TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Tuesday, August 22, 1939 — 3.30 to 6.30 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1939.

The minimum time requirement is five recitations a week for half a school year, or the equivalent. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1939 is required.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two groups, including at least *two* questions from each group.

Plane Trigonometry

See instructions for groups II and III on page 1.

Answer five questions from groups II and III, including at least two questions from each group.

Group II

Answer at least two questions from this group.

21 A tower CD is surmounted by a vertical 30-foot electric sign DA. From B, a point on a level with C, the base of the tower, the angles of elevation of the top and bottom of the sign are 41° 30′ and 36° 30′ respectively. How high is the tower? [10]

22 Three sides of a triangle are 83.4, 71.5 and 55.1. Using logarithms, find the angle opposite the shortest side. [10]

23 An airplane was observed to be flying horizontally away from the observer at a height of 8430 feet. From a certain point its angle of elevation was 33° 40′. One minute later from the same point the angle of elevation was 20° 50′. If the observer and the line of flight were in the same vertical plane, find, in *feet per minute,* the speed of the airplane. [10]

24 The distances from point A to two points B and C on opposite sides of a hill are 2430 feet and 3050 feet respectively. Angle BAC is 50°. Find the angle that AB would make with a tunnel from B to C. [10]

Group III

Answer at least two questions from this group.

25 Prove that in an acute triangle the square of any side is equal to the sum of the squares of the other two sides minus twice the product of these two sides times the cosine of the included angle. [10]

26 *a* Make a table of values of x and y for the equation $y = 2 \cos x$ as x varies from 0° to 360° inclusive, in intervals of 30°. [5]

b Using these values, draw the graph of the equation. [5]

27 Solve the equation $\tan A + 2 = \cot A$ for all values of A between 0° and 180°. [10]

28 In right triangle ABC, prove

a $\cos 2A = \sin (B - A)$ [5]

b $K = \dfrac{c^2 \sin 2A}{4}$, in which K is the area of the triangle. [5]

*29 Using DeMoivre's Theorem, find the value of $(1 + i)^{10}$ [10]

* This question is based on one of the optional topics in the syllabus.

PLANE TRIGONOMETRY

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 In triangle ABC, $a = 2$, $b = 3$ and $C = 60°$; find c. [Answer may be left in radical form.] 1.....

2 Two sides and the included angle of a triangle are respectively 4, 7 and 150°; find the area of the triangle. 2.....

3 If $\sin A = \frac{8}{17}$ and A is obtuse, what is the value of $\cos A$? 3.....

4 Express $\sec^2 A$ in terms of $\sin A$. 4.....

5 Express $\sin^2 \frac{1}{2}A$ in terms of $\cos A$. 5.....

6 Find $\log \sin 40° 38'$ 6.....

7 Find $\log \cos 35° 36'$ 7.....

8 Find, correct to the *nearest minute,* the positive acute angle A for which $\log \sin A = 9.9394 - 10$ 8.....

9 Express $\sin (-205°)$ as a function of a positive acute angle. 9.....

10 From a point 1200 feet directly above an airport, the angle of depression of an armory is 20°. If the armory and airport are on level ground, what is the distance between them? 10.....

11 If a ship sailed from a certain point a distance of 20 miles in the direction N. 24° 30′ W., how far north did it sail? 11.....

12 Through how many radians does the hour hand of a clock pass in 12 hours? 12.....

13 What is the value of $\tan (\tan^{-1} \frac{1}{2})$? 13.....

14 Find the value of x between 90° and 180° which satisfies the equation $\sin^2 x = 3 \cos^2 x$ 14.....

15 Write the smallest positive value of A that satisfies the equation $\cot 2A = 1$ 15.....

16 What is the numerical value of $\csc^2 A(1 - \cos^2 A)$? 16.....

17 If $\cos A = k$, what is the value of $\cos 2A$ in terms of k? 17.....

18 If $\tan A = \frac{2}{3}$, what is the value of $\tan(45° - A)$? 18.....

Directions (questions 19–20) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a, b, c or d.

19 If $\sin (180° - A) = m$, and $\csc A = n$, then (a) $m = n$, (b) $m = -n$, (c) $m = \frac{1}{n}$ or (d) $m = -\frac{1}{n}$ 19.....

20 As the cosecant of a positive angle decreases from ∞ to 1, the value of its cosine ranges from (a) 0 to —1, (b) 1 to 0, (c) —1 to 0 or (d) 0 to 1. 20.....