PLANE TRIGONOMETRY

Tuesday, August 24, 1937 — 3.30 to 6.30 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1937.

The minimum time requirement is five recitations a week for half a school year, or the equivalent. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1937 is required.

In this examination the customary lettering is used. *A*, *B* and *C* represent the angles of a triangle *ABC*; *a*, *b* and *c* represent the respective opposite sides. In a right triangle, *C* represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

See instructions for group II on page 1.

Group II
Answer question 21 and four of the others.

21 In triangle ABC, $a = 42.31$, $c = 49.82$ and $B = 81° \; 22'$; find C by copying and completing the following outline: [10]

$$\tan \tfrac{1}{2}(C - A) = \frac{c - a}{c + a} \tan \tfrac{1}{2}(C + A)$$

$c - a = \ldots\ldots$ $\log (c - a) = \ldots\ldots$

$c + a = \ldots\ldots$ $\log \tan \tfrac{1}{2}(C + A) = \ldots\ldots$

$C + A = \ldots\ldots$ $\text{colog} (c + a) = \ldots\ldots$

$\log \tan \tfrac{1}{2}(C - A) = \ldots\ldots$

$\tfrac{1}{2}(C - A) = \ldots\ldots$

$\tfrac{1}{2}(C + A) = \ldots\ldots$

$C = \ldots\ldots$

22 a Starting with the formula for sin $(x + y)$, derive the formula for sin $2x$. [4]

b Starting with the formulas for sin $(x + y)$ and cos $(x + y)$, derive the formula for tan $(x + y)$. [6]

23 a Solve the following equation for positive values of x less than 360° and check *one* answer:
$3 \cos x = 2 \sin^2 x$ [5, 1]

b Express $\dfrac{\cos x}{1 - \sin x}$ as an equivalent fraction whose denominator is cos x. [4]

24 To find the distance from A, a point on shore, to C, a rock on an island, a distance AB along the shore, and angles CAB and CBA were measured. $AB = 650$ feet, angle $CAB = 77° \; 44'$ and angle $CBA = 58° \; 22'$; find, correct to the *nearest foot*, the distance from A to the rock. [2, 8]

25 a A balloon is acted upon by two forces — a vertical force which causes it to rise and the horizontal force of the wind. If a balloon rises obliquely to a vertical height of 612 meters in 3 minutes and a meteorologist at the point of departure observes that its angle of elevation is 21° 51', what is the velocity of the wind? [8]

b If the direction of the balloon's flight is 135° east of north, from what direction does the wind blow? [2]

26 From two points in a horizontal line with the base of a tower, the angles of elevation of the top of the tower were observed to be 62° 52' and 31° 28' respectively. If the first point is 400 feet nearer the base of the tower than the second, find, correct to the *nearest foot*, the height of the tower. [3, 7]

27 Draw the graphs of $y = \cos x$ and $y = \sin \tfrac{1}{2}x$ from $x = 0°$ to $x = 180°$ in intervals of 30° and from the graphs determine a value of x common to the two equations. [3, 5, 2]

*28 Express in polar form the result of multiplying $\sqrt{3} + i$ by $1 + i\sqrt{3}$ [10]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school..Name of pupil..

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 A tunnel extends into the earth at an angle of 15° 23′ with the surface. A man descends 300 feet along this tunnel. Assuming that the ground is level, find, correct to the *nearest foot,* how far he is below the surface. *Ans*............

2 Express 225° in radians. [Answer may be left in terms of π.] *Ans*............

3 Find the numerical value of cot 37° 32′ *Ans*............

4 If tan $x = a$, express tan $2x$ in terms of a. *Ans*............

5 Express the tangent of an acute angle x in terms of sec x. *Ans*............

6 Express cos 238° as a function of an acute angle less than 45°. *Ans*............

7 Find the value of sin $(-67°)$ *Ans*............

8 As a positive angle increases in the fourth quadrant, does its secant increase or decrease? *Ans*............

9 Find all values of A between 0° and 360° that satisfy the equation cos $A = -$ sin A *Ans*............

10 At a certain point, the captain of a ship sailing south observed a lighthouse due east. After sailing 6 miles farther south, he found the direction of the light to be N. 60° E. How far was the ship from the lighthouse at the time of the second observation? *Ans*............

11 Write the formula for tan $\frac{1}{2} A$ in terms of s, a, b and c. *Ans*............

12 If cot $x = 1$, what is the value of cos $2x$? *Ans*............

13 If cos $A = -$ .28, and A is an obtuse angle, find sin $\frac{1}{2}A$ without using tables. *Ans*............

14 Find, correct to the nearest *square inch,* the area of triangle ABC if $a = 4$ inches, $b = 5$ inches and $C = 63°$ *Ans*............

15 Log cos $\dfrac{C}{2} = 9.9167 - 10$; find C correct to the *nearest minute* if C is an acute angle. *Ans*............

16 Find, correct to the *nearest pound,* the resultant of forces of 8 pounds and 4 pounds acting at right angles to each other on the same body. *Ans*............

17 If the eye of an observer is 6 feet from one end of a yardstick and 8 feet from the other, what is the cosine of the angle subtended by the yardstick? [Answer may be left as a common fraction.] *Ans*............

[3] [OVER]

Directions (questions 18–20) — In each of these questions *one* statement is incorrect. Indicate the incorrect statement by writing (a), (b) or (c) in the space provided.

18 If $x = \sin^{-1} \frac{1}{2}$, then (a) $x = 60°$, (b) $\sin x = \frac{1}{2}$ or (c) $x = \dfrac{\pi}{6}$ *Ans*............

19 If $\log x = m$, then (a) colog $x = -m$, (b) colog $x = 10 - m$ or (c) colog $x + \log x = 0$ *Ans*............

20 In every triangle, (a) $\sin (A + B) = \sin C$, (b) $\sin (A + B) = \cos C$ or (c) $\sin \frac{1}{2} (A + B) = \cos \frac{1}{2} C$ *Ans*............

[4]