The University of the State of New York

281st High School Examination

# PLANE TRIGONOMETRY

Thursday, June 19, 1941 — 9.15 a. m. to 12.15 p. m., only

## Instructions

*Do not open this sheet until the signal is given.*

### Part I

*This part is to be done first and the maximum time allowed for it is one and one half hours.* Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

### Parts II and III

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. $A$, $B$ and $C$ represent the angles of a triangle $ABC$; $a$, $b$ and $c$ represent the respective opposite sides. In a right triangle, $C$ represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two parts, including at least *two* questions from each part.

**Fill in the following lines:**

Name of school.....Name of pupil.....

**Detach this sheet and hand it in at the close of the one and one half hour period.**

## Part I

**Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.**

1 Find the number of degrees in $\frac{5}{9}\pi$ radians. 1.....

2 Express cos (—350°) as a function of a positive acute angle. 2.....

3 Find the numerical value of tan 240°. [Answer may be left in radical form.] 3.....

4 Find the positive value of sin $(\tan^{-1}\frac{8}{15})$ 4.....

5 Find, correct to the *nearest thousandth*, the number whose logarithm is 0.7111. 5.....

6 If log sin $x = 9.8355 - 10$ and $x$ is a positive acute angle, find the value of $x$ correct to the *nearest minute*. 6.....

7 Find the value of tan 31° 23′. 7.....

8 Express tan $(x - y)$ in terms of tan $x$ and tan $y$. 8.....

9 In which quadrant does an angle lie if its sine is positive and its tangent is negative? 9.....

10 Express $\cot^2 A$ as a function of sin $A$. 10.....

11 If $\sin x = \frac{3}{5}$, find the value of cos $2x$. 11.....

12 In triangle $ABC$, $a = 10$, $\sin A = \frac{2}{3}$ and $\sin B = \frac{3}{5}$; find $b$. 12.....

13 In triangle $ABC$, $a = 5$, $b = 11$, $c = 13$; find cos $B$. 13.....

14 Given $\cos x = \frac{1}{9}$; find the positive value of $\sin \frac{1}{2}x$. 14.....

15 Find the value of $x$ between 180° and 270° which satisfies the equation $\tan^2 x - 1 = 0$ 15.....

16 In triangle $ABC$, $C = 90°$, $c = 110$, $A = 42°$; find $b$ correct to the *nearest tenth*. 16.....

17 If $\tan A = x$, express tan $2A$ as a function of $x$. 17.....

18 How many different triangles can be formed in which $a = 10$, $b = 11$, $A = 27°$? 18.....

19 As angle $A$ increases from 90° to 180°, cos $A$ $(a)$ increases from —1 to 0, $(b)$ decreases from 0 to —1, $(c)$ decreases from 1 to 0. Which is correct $(a)$, $(b)$ or $(c)$? 19.....

20 The minimum value of 2 sin $x$ is $(a)$ 0, $(b)$ —1, $(c)$ —2. Which is correct $(a)$, $(b)$ or $(c)$? 20.....

**See instructions for parts II and III on page 1.**

*Answer five questions from parts II and III, including at least two questions from each part.*

## Part II

**Answer at least two questions from this part.**

21 *a* Prove the identity: $\tan A = \dfrac{\sin A + \sin 2A}{1 + \cos A + \cos 2A}$ [5]

*b* Find, correct to the *nearest degree*, the value of $x$ between 0° and 90° which satisfies the equation: $1 - 3\cos^2 x + 5 \sin x = 0$ [5]

22 Derive the formula: $\dfrac{a+b}{a-b} = \dfrac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$ [10]

23 At a distance $a$ from the foot of a tower, the angle of elevation $x$ of the top is the complement of the angle of elevation of the top of a flagstaff on the tower. Show that the length of the flagstaff is $a(\cot x - \tan x)$. [10]

24 *a* Draw the graph of $y = \cos 2x$ as $x$ varies from 0° to 90° inclusive at intervals of 15°. [8]

*b* On the graph made in answer to *a*, draw the ordinate that represents the cosine of 45°. [2]

*25 *a* Express in polar form $\sqrt{3} + i$. [4]

*b* With the aid of De Moivre's theorem, show that $[2(\cos 60° + i \sin 60°)]^5$ equals $16 - 16i\sqrt{3}$ [6]

## Part III

**Answer at least two questions from this part.**

26 In triangle $ABC$, $AB = 50$, $A = 71°$, $C = 49°$; find the length of the altitude on $AB$ correct to the *nearest integer*. [10]

27 In triangle $ABC$, $a = 61.4$, $b = 80.5$, $c = 70.1$; find $C$ correct to the *nearest minute*. [10]

28 A radio station $B$ is 300 miles south of station $A$. A ship $C$ is south 41° 10′ east of $A$, and south 68° 40′ east of $B$. Find, correct to the *nearest mile*, the distance from $C$ to $B$. [10]

29 A body is acted upon by two forces of 320 and 205 pounds acting at an angle of 80°. Find, correct to the *nearest degree*, the angle at which the resultant is inclined to the 320-pound force. [10]

* This question is based on one of the optional topics in the syllabus.