The University of the State of New York

275TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, June 22, 1939 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two groups, including at least *two* questions from each group.

Fill in the following lines:

Name of school..... Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–15) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 Expressed in terms of π, the number of radians in an angle of 45° is ... 1.....

2 The positive value of cos $(\tan^{-1} \frac{5}{12})$ is ... 2.....

3 The logarithm of 6.573 is ... 3.....

4 The numerical value of sin (—210°) is ... 4.....

5 The value of cos 22° 42′ is ... 5.....

6 The value of log tan 52° 24′ is ... 6.....

7 The formula for tan $2A$ in terms of tan A is tan $2A$ = ... 7.....

8 If the value of cos x is .82, the positive value of sin $\frac{1}{2} x$ is ... 8.....

9 In $\triangle ABC$, $b = 6$, $c = 4$ and cos $A = \frac{1}{16}$; then a = ... 9.....

10 In $\triangle ABC$, $a = 4$, $b = 10$, $B = 150°$; then sin A = ... 10.....

11 In $\triangle ABC$, $a = 3$, $b = 2$, $A + B = 120°$; then tan $\frac{1}{2}(A - B)$ = ... [Answer may be left in radical form.] 11.....

12 The value of x between 180° and 270° which satisfies the equation $4 \sin^2 x - 3 = 0$ is ... 12.....

13 An inclined driveway is to be built from the ground to a second-story doorway. The doorway is 12 feet above the ground and the angle of elevation of the driveway is to be 24°. The distance, correct to the *nearest foot*, from the base of the building to the beginning of the driveway will be ... 13.....

14 If cos $A = \frac{12}{13}$, sin $B = \frac{3}{5}$ and A and B are positive acute angles, the value of cos $(A + B)$ is ... 14.....

15 $\text{Tan}^2 A$ expressed in terms of sin A is $\tan^2 A$ = ... 15.....

Directions (questions 16–20) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a, b or c.

16 Cos $(180° - A)$ equals (a) cos A, (b) — cos A or (c) — sin A. 16.....

17 In the right triangle ABC, the statement sin $2A$ = sin $2B$ is (a) always true, (b) sometimes true or (c) never true. 17.....

18 The number of different triangles that can be formed in which $A = 40°$, $c = 50$ and $a = 51$ is (a) 2, (b) 1 or (c) 0. 18.....

19 When drawn on the same set of axes, the graphs of y = sin x and y = tan x, for values of x from 0° to 360° inclusive, intersect (a) once, (b) three times or (c) five times. 19.....

20 The maximum value of 3 sin $\frac{1}{2} A$ is (a) $\frac{1}{2}$, (b) 1 or (c) 3. 20.....

See instructions for groups II and III on page 1.

Answer <u>five</u> questions from groups II and III, including at least two questions from each group.

Group II

Answer at least two questions from this group.

21 *a* Find, correct to the *nearest degree*, the positive acute angle x that satisfies the equation $2 \tan x - 3 \cot x = 1$ [5]

b Prove the identity: $\dfrac{\cos B + \sin B}{\cos B - \sin B} = \dfrac{\sin 2B + 1}{\cos 2B}$ [5]

22 Starting with the formula for the law of sines, derive the formula for the law of tangents. [10]

23 Given triangle ABC with h the altitude on side c; prove: $h = \dfrac{c}{\cot A + \cot B}$ [Give proof for the acute triangle only.] [10]

24 *a* Draw the graph of the equation $y = \cos x$ as x varies from 0° to 180° inclusive in intervals of 30°. [4]

b Using the same set of axes as in *a*, draw the graph of the equation $y = \tan x$ as x varies from 0° to 180° inclusive in intervals of 30°. [5]

c With the aid of the graphs drawn in answer to *a* and *b*, find, correct to the *nearest degree*, *one* value of x for which $\cos x = \tan x$. [1]

*25 Using the relationships between rectangular coordinates and polar coordinates,

a Transform the equation $x^2 + y^2 = 25$ into a corresponding equation in polar coordinates [6]

b Transform the equation $\tan \theta = 1$ into the corresponding equation in rectangular coordinates [4]

Group III

Answer at least two questions from this group.

26 In $\triangle ABC$, $AB = 42.5$, $AC = 23.6$ and $BC = 31.9$; find, correct to the *nearest minute*, the smallest angle of the triangle. [10]

27 The lengths of two sides of a triangular meadow are 48 rods and 35 rods, and these sides meet at an angle of 106°. Find, correct to the *nearest acre*, the area of the meadow. [One acre = 160 square rods] [10]

28 The radio operator on a Coast Guard cutter received an SOS call and found that the endangered ship was located N 58° 20′ E. The operator on a second Coast Guard cutter, 82 miles due north of the first, found that the call came from a position S 69° 10′ E. Find, correct to the *nearest mile*, the distance from the source of the SOS call to the nearer Coast Guard cutter. [10]

29 A body is acted upon by two forces of 210 pounds and 28 pounds. These forces act at an angle of 76° with each other. Find, correct to the *nearest minute*, the angle at which the resultant is inclined to the force of 210 pounds. [10]

*This question is based on one of the optional topics in the syllabus.