The University of the State of New York

272D HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, June 23, 1938 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. *A*, *B* and *C* represent the angles of a triangle *ABC*; *a*, *b* and *c* represent the respective opposite sides. In a right triangle, *C* represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two groups, including at least *two* questions from each group.

See instructions for groups II and III on page 1.

Answer five questions from groups II and III, including at least two questions from each group.

Group II
Answer at least two questions from this group.

21 *a* Starting with the formula for the cosine of the sum of two angles, derive the formula
$$\cos 2A = 1 - 2 \sin^2 A \qquad [6]$$

b Prove the identity $\dfrac{\sec A}{\cot A + \tan A} = \sin A$ [4]

22 Solve $4 \cos^2 x - 5 \sin x + 2 = 0$ for the positive value of x less than $90°$. [Express your answer correct to the *nearest minute.*] [10]

23 *a* Represent the sine, cosine and tangent of an angle in the third quadrant as line segments. Give a full description of your diagram. [7]

b For each function indicate the line segment representing it. [3]

24 Plot the graph of the equation $y = \tan x$ as x varies from $0°$ to $180°$ inclusive in intervals of $30°$. [10]

*25 With the aid of DeMoivre's Theorem

a Find the positive square root of $2\sqrt{2} + 2i\sqrt{2}$ [Leave answer in polar form.]   [6]

b Prove: $(\cos 30° + i \sin 30°)^3 = i$ [4]

Group III
Answer at least two questions from this group.

26 Two angles of a triangle are $25°$ and $70°$ and the longest side is 56 feet. Find the shortest side correct to the *nearest foot.* [10]

27 A man on a cliff 1000 feet above sea level observes two ships due west. The angles of depression of the ships are $56°$ and $32°$. Find, correct to the *nearest foot,* the distance between the ships. [10]

28 The sides of a triangle are 36.8, 42.7 and 48.5; find, correct to the *nearest minute,* the largest angle of the triangle. [10]

29 A tunnel extends in a straight line between two points, A and B, on opposite sides of a hill. At a point C 820 feet from A and 640 feet from B the angle subtended by the line AB is $78°$. Find, correct to the *nearest foot,* the length of the tunnel. [10]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school..Name of pupil...

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express in degrees $\frac{7}{6}\pi$ radians.   1...............

2 Express $\log \dfrac{a^2}{b}$ in terms of $\log a$ and either $\log b$ or $\mathrm{colog}\, b$.   2...............

3 Express $\sin^2 \frac{1}{2} A$ in terms of $\cos A$.   3...............

4 Find $\log \tan 24° 36'$.   4...............

5 Find, correct to the *nearest minute,* the positive acute angle A for which $\cos A = 0.8642$ 5...............

6 Find the logarithm of 0.5663   6...............

7 In triangle ABC, $b = 4$, $c = 6$, $B = 30°$; find $\sin C$.   7...............

8 In triangle ABC, $A = 60°$, $b = 5$, $c = 8$; find a.   8...............

9 In triangle ABC, $a = 15$, $b = 5$, $A - B = 60°$; find the value of $\tan \frac{1}{2} (A + B)$. [Answer may be left in radical form.]   9...............

10 For what angle greater than $45°$ and less than $360°$ does $\sin A$ equal $\cos A$?   10...............

11 Find the positive acute angle x which satisfies the equation
$$2 \sin x - \sqrt{3} = 0$$ 11...............

12 Angle $A = \csc^{-1} \dfrac{\sqrt{5}}{2}$; write the positive value of $\tan A$.   12...............

13 The area of a parallelogram $ABCD$ is 60 square inches, angle $A = 30°$ and $AB = 12$ inches; find AD.   13...............

14 Is $(\sin A + \cos A)^2 = 1$ an identity? [Answer *yes* or *no*.]   14...............

15 If x and y are both acute angles such that $\tan x = \frac{1}{2}$ and $\tan y = \frac{1}{3}$, find the value of $\tan (x + y)$.   15...............

Directions (questions 16–20) — In each of the following indicate which is correct, (a), (b) or (c):

16 Cos $180°$ equals (a) $+1$, (b) 0 or (c) -1.   16...............

17 Sin $(180° - x)$ equals (a) $\sin x$, (b) $-\sin x$ or (c) $-\cos x$.   17...............

18 Cos $(-x)$ equals (a) $-\cos x$, (b) $\cos x$ or (c) $-\sin x$.   18...............

19 Sin $2A = 2 \sin A$, (a) for all values of A, (b) when $A = 180°$ or (c) when $A = 90°$ 19...............

20 Sin $(45° + x)$ equals (a) $\dfrac{\cos x}{\sqrt{2}} - \dfrac{\sin x}{\sqrt{2}}$, (b) $\dfrac{1}{\sqrt{2}} + \sin x$ or (c) $\dfrac{\cos x}{\sqrt{2}} + \dfrac{\sin x}{\sqrt{2}}$ 20...............

[3]