The University of the State of New York

266TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, June 18, 1936 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express 75° in radian measure. [Answer may be left in terms of π.] *Ans*.....

2 Express tan 127° as a function of a positive angle less than 45°. *Ans*.....

3 Express tan $2A$ in terms of tan A. *Ans*.....

4 Write the formula for cos $(A - B)$. *Ans*.....

5 Find the logarithm of 0.4312 *Ans*.....

6 Find the numerical value of sin 48° 12′ *Ans*.....

7 Find, correct to the *nearest minute*, the acute angle A, when $\log \cos A = 9.5655 - 10$ *Ans*.....

8 The vertex angle of an isosceles triangle is 64° and the equal sides are 10. Find the area of the triangle correct to the *nearest integer*. *Ans*.....

9 In a circle of radius 10″ a chord subtends a central angle of 48°; find the length of the chord correct to the *nearest inch*. *Ans*.....

10 Find the positive value of $\tan(\sin^{-1}\frac{\sqrt{2}}{2})$ *Ans*.....

11 Is $\sin 2A = \frac{2\tan A}{\sec^2 A}$ an identity? *Ans*.....

12 Find the numerical value of $\tan\frac{1}{2}(A - B)$ if $a + b = 15$, $a - b = 5$ and $\tan\frac{1}{2}(A + B) = 1$ *Ans*.....

13 What value of x between 0° and 180°, other than 30° and 150°, satisfies the equation $\sin 2x = \cos x$? *Ans*.....

14 In triangle ABC, $C = 90°$, $c = 51$, $\sin B = \frac{8}{17}$; find b. *Ans*.....

15 In triangle ABC, $\frac{a}{b} = \frac{\sqrt{2}}{2}$ and $A = 30°$; find acute angle B. *Ans*.....

16 In triangle ABC, $A = 60°$, $b = 4$, $c = 5$; find a. [Answer may be left in radical form.] *Ans*.....

Directions (questions 17–20) — Indicate the correct answer to each of the questions below by writing (a), (b) or (c) in the space provided.

17 If $\sin x = P$ and $\csc x = Q$, then (a) $P = Q$, (b) $P = -Q$ or (c) $P = \frac{1}{Q}$ *Ans*.....

18 If $\cos A = \sin B$, then (a) $A = 90° - B$, (b) $A = 90° + B$ or (c) $A = 180° - B$ *Ans*.....

19 If $\log 4.72$ equals m, then $\log 472$ equals (a) $100m$, (b) $2m$ or (c) $m + 2$ *Ans*.....

20 State which one of the following statements is never true: (a) $\sin x + \cos x = 2$, (b) $\sin x \tan x = \cos x$, (c) $\sin x = \tan x$ *Ans*.....

See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

21 *a* Starting with the formula $\cos 2A = \cos^2 A - \sin^2 A$, derive the formula expressing $\sin A$ in terms of $\cos 2A$. [6]

b Show that $\sin^2 A + \cos^2 A = 1$ [4]

22 *a* Prove the identity: $\dfrac{\sin (x + y)}{\sin (x - y)} = \dfrac{\tan x + \tan y}{\tan x - \tan y}$ [5]

b Solve the following equation for all values of x between $0°$ and $360°$:
$4 \cos^2 x - 8 \sin x = 7$ [5]

23 *a* In any right triangle ABC, (1) $\tan \frac{1}{2} A = \dfrac{c - b}{a}$, (2) $\tan \frac{1}{2} A = \dfrac{a}{b + c}$; prove either (1) or (2). [5]

b For the case in which all the angles of triangle ABC are acute, show that
$a = b \cos C + c \cos B$ [5]

24 *a* Using the same set of axes, plot the graph of $y = \sin x$ and the graph of $y = \sin 2x$, as x varies from $0°$ to $360°$ in intervals of $45°$. [8]

b With the aid of the graphs made in answer to *a* obtain the value of x between $0°$ and $180°$ common to both equations. [2]

*25 Find the *modulus* and the *amplitude* of *each* of the following complex numbers: (*a*) $\sqrt{3} + i$, (*b*) $2 + 2i\sqrt{3}$, (*c*) $(\sqrt{3} + i)(2 + 2i\sqrt{3})$ [3, 3, 4]

Group III

Answer two questions from this group.

26 A parallelogram with sides of 7 inches and 6 inches has an area of 21 square inches. Find the length of the shorter diagonal correct to the *nearest tenth of an inch.* [10]

27 An airplane is flying horizontally due east at a height of 9750 feet. From a certain point its angle of elevation is observed to be $32°\ 20'$. One minute later its angle of elevation is $21°\ 40'$. Find the distance covered during the one-minute interval, assuming that the observer and the line of flight are in the same vertical plane. [10]

28 In triangle ABC, $A = 53°\ 10'$, $a = 16.7$, $b = 13.9$; find B and c. [10]

* This question is based on one of the optional topics in the syllabus.