The University of the State of New York

263D HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, June 20, 1935 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Given $\log \cos A = 9.8898 - 10$; find, correct to the *nearest minute*, the value of A if A is a positive acute angle. *Ans*.....

2 Find $\log \tan 31° 12'$ *Ans*.....

3 $A = \frac{7\pi}{6}$ radians; express A in degrees. *Ans*.....

4 In a circle of radius 7 inches, an arc subtends a central angle of two radians; find the length of the arc in inches. *Ans*.....

5 If A, B and C are the angles of a triangle, write $\sin (B + C)$ as a function of A. *Ans*.....

6 Write as a single term the sum of $\log 1000 + \log \tan 45° + \log .1$ *Ans*.....

7 Given $A = \sin^{-1} \frac{4}{5}$, where A is an acute angle; write the numerical value of $\sin 2A$. *Ans*.....

8 Express $\tan 2x$ in terms of $\tan x$. *Ans*.....

9 In triangle ABC, $A = 30°$, $B = 45°$, $a = 2$; find b. *Ans*.....

10 In triangle ABC, $a = \sqrt{3}$, $b = 1$, $c = 2$; find A. *Ans*.....

11 In triangle ABC, $\tan \frac{1}{2}(A + B) = \frac{3}{4}$, $\tan \frac{1}{2}(A - B) = \frac{1}{2}$, $a + b = 18$; find $a - b$. *Ans*.....

12 In triangle ABC, $a = 7$, $c = 8$, $\sin B = \frac{1}{4}$; find the area of triangle ABC. *Ans*.....

13 For what positive value of A less than 90° is $2 \sin A \cos A$ equal to 1? *Ans*.....

14 Express $\sin (-230°)$ as a function of a positive angle less than 45°. *Ans*.....

15 Given $\tan x = \frac{1}{2}$ and $\tan y = \frac{1}{3}$; find the value of $\tan (x - y)$. *Ans*.....

16 Express $\cos A$ in terms of $\tan A$, where A is an angle in the first quadrant. *Ans*.....

17 As the sine of an angle decreases from 1 to 0, the cosecant of the angle (*a*) decreases from 1 to −1, (*b*) increases from 0 to 1 or (*c*) increases from 1 to ∞. Which is correct, (*a*), (*b*) or (*c*)? *Ans*.....

18 Express $\sin 10°$ in terms of $\cos 20°$. *Ans*.....

19 What is the greatest possible value $\frac{1}{3} \sin 3A$ can have? *Ans*.....

20 What is the value of x between 0° and 180° where the curves $y = \sin x$ and $y = \cos x$ intersect? *Ans*.....

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See instructions for groups II and III on page 1.

Group II

Answer two questions from this group.

21 *a* Derive the formula for $\sin (A + B)$, where A, B and $(A + B)$ are positive acute angles. [8½]

b Derive the formula for $\cos 2A$ in terms of $\cos A$, starting with the formula for $\cos (x + y)$. [4]

22 Find, correct to the *nearest minute*, the value of x between 0° and 60° that satisfies the equation $8 \cos 2x - 4 (1 - 2 \sin^2 x) = 1$ [12½]

23 *a* Solve for x the equation $\sec^2 x - \tan x = 7$, expressing your answer as inverse functions. [6]

b Prove the identity: $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$ [6½]

*24 Sketch the graph of $r = 10 (1 - \sin \theta)$, using the following values for θ: 0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°. [12½]

Group III

Answer two questions from this group.

25 Two straight railroad tracks intersect at J. Two trains traveling at rates of 40 and 48 miles an hour leave J at noon. At 2.30 p. m. the trains are 60 miles apart. Find, correct to the *nearest minute*, the angle at which the tracks intersect. [12½]

26 At two points 124 feet apart on a horizontal line running due east from the base of a building, the angles of elevation of the top of the building are 41° 15′ and 24° 20′; find the height of the building. [12½]

27 Two forces, one of 7.24 pounds and the other of 4.48 pounds, act on a body at an angle of 68° 20′; find the magnitude and the direction of the resultant of these forces. [12½]

*28 Using De Moivre's Theorem, solve completely the equation $x^3 - 1 = 0$ [12½]

* This question is based on one of the optional topics in the syllabus.