The University of the State of New York

239TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Wednesday, June 15, 1927 — 1.15 to 4.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement for plane trigonometry is five recitations a week for half a school year, or the equivalent.

Answer seven questions, including three from group I and four from group II.

A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

In the examination in plane trigonometry the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

Group I

Answer three questions from this group.

1 Two points A and B on a horizontal plane are due south of a mountain top, A being nearer the mountain. AB is 1200 feet and the angles of elevation of the top of the mountain at A and B are 12° 18′ and 10° 30′ respectively. How high is the mountain above the horizontal plane? [16]

2 Two points A and B are separated by a grove but both are visible from a third point C. If A is 300 feet due north of C, and B is 200 feet northwest of C, find angles ABC and BAC. [16]

3 The sides of a quadrangular field $ABCD$ are $AB = 400$ feet, $BC = 958.2$ feet, $CD = 1276.6$ feet, $DA = 300$ feet. If angle A is a right angle, find angle C. [16]

4 A tree stands at a point A on the bank of a river. To measure its height from the opposite shore, a line 60 feet long is laid off along the shore from the point C directly opposite A to a point B, the line CB being at right angles to CA. Angle ABC is measured and found to be 58° 46′. At C the angle of elevation of the top of the tree is found to be 27° 34′ 24″. How high is the tree? [16]

Group II

Answer four questions from this group.

5 Derive the formulas for $\cos 2A$ and $\cos \frac{1}{2}A$, each in terms of $\cos A$. [13]

6 Prove the following identities:

a $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$ [6]

b $\sin (x+y)\cos y - \cos (x+y)\sin y = \sin x$ [7]

7 *a* Solve the equation $\cot x + 2\cos x = 0$ for all values of x between 0° and 360°. [9]

b How many radians are there in an angle of 125°? [2]

c If the radius of a circle is 10 inches, what is the length in inches of an arc that subtends a central angle of 125°? [2]

8 A flagstaff stands on the edge of a vertical cliff. At a point in the horizontal plane through the foot of the cliff and at a distance d from the foot, the angle of elevation of the top of the cliff is $m°$ and the angle subtended by the flagstaff is also $m°$.

a Find the height of the cliff in terms of d and $\tan m°$. [4]

b Find the height of the flagstaff in terms of d and $\tan m°$. [9]

9 *a* Construct a table of values for $y = \cos x$ at intervals of 30° as x varies from 0° to 360°. [5]

b Plot the graph of $y = \cos x$ [7]

c How does the graph show the sign of $\cos x$ in each quadrant? [1]

10 *a* If $x = \sin^{-1}\frac{3}{5}$, x being an angle in the second quadrant, find $\tan \frac{1}{2}x$ and $\tan 2x$. [7]

b Express the following as functions of positive acute angles: $\sin 212°$, $\cos (-280°)$, $\tan \frac{6\pi}{5}$ [6]