The University of the State of New York

283d High School Examination

PLANE TRIGONOMETRY

Thursday, January 22, 1942—9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II and III

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two parts, including at least *two* questions from each part.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express 270° in radian measure. [Answer may be left in terms of π.] 1.....

2 Express sin (—230°) as a function of a positive angle less than 45°. 2.....

3 Find the value of $\cos \frac{2\pi}{3}$ 3.....

4 If $x = \sin^{-1} \frac{3}{5}$ and is in the second quadrant, find $\tan x$. 4.....

5 Find, correct to the *nearest thousandth*, the number whose logarithm is 0.4832 5.....

6 If $\log \cos x = 9.6056 - 10$, find, correct to the *nearest minute*, the smallest positive value of x. 6.....

7 Find the value of cot 33° 24′ 7.....

8 Express $\cos^2 \frac{1}{2}x$ in terms of $\cos x$. 8.....

9 Express $\sin (90 + A)$ as a function of A. 9.....

10 In triangle ABC, $a = 2b$ and $\tan \frac{1}{2}(A + B) = 1$; find the value of $\tan \frac{1}{2}(A - B)$. 10.....

11 In triangle ABC, $a = 6$, $c = 8$ and $\cos B = \frac{3}{8}$; find b. 11.....

12 How many different triangles can be formed in which $a = 14$, $b = 12$ and $A = 30°$? 12.....

13 As angle A increases from 90° to 180°, which function of A decreases from 1 to 0? 13.....

14 The legs of an isosceles triangle are each 20 inches long and its vertex angle is 30°; find, correct to the *nearest inch*, the altitude to the base. 14.....

15 Is the equation $2 \sin^2 x + 5 \sin x = 3$ satisfied when x has the value 30°? 15.....

16 Write the formula involving a, b and $\sin A$ which can be used to find B in triangle ABC. 16.....

17 What function of $(A + B)$ is $\sin A \cos B + \cos A \sin B$? 17.....

18 Is the following statement *true* or *false*: "If an angle is negative, all of its functions are negative"? 18.....

Directions (questions 19–20) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a, b or c.

19 The statement $\sin 2A = 2 \sin A$ is (a) always true, (b) never true or (c) sometimes true. 19.....

20 Which of the following statements is never true: (a) $\frac{\sin x}{\cos x} = 2$, ($b$) $\sin x + \cos x = 1$ or (c) $\sin 3x = 3$? 20.....

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See instructions for parts II and III on page 1.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from this part.

21 a Prove the identity: $\sin 2A\ (\sec A + \csc A) \equiv 2(\sin A + \cos A)$ [4]

b Find, correct to the *nearest degree*, the positive acute angle which satisfies the equation $3\cos^2 x + 8\sin x - 7 = 0$ [6]

22 Derive the formula for $\cos(x + y)$ in which x, y and $x + y$ are positive acute angles. [10]

23 Below is a proof for the theorem: The bisector of an angle of a triangle divides the opposite side into segments which are proportional to the adjacent sides. The statements refer to triangle ABC in which the bisector of angle A intersects BC in D. Give a reason for each statement. [10]

a $\dfrac{BD}{AD} = \dfrac{\sin \frac{1}{2}A}{\sin B}$

b $\dfrac{DC}{AD} = \dfrac{\sin \frac{1}{2}A}{\sin C}$

c $\dfrac{BD}{DC} = \dfrac{\sin C}{\sin B}$

d $\dfrac{AB}{AC} = \dfrac{\sin C}{\sin B}$

e $\dfrac{BD}{DC} = \dfrac{AB}{AC}$

24 a Draw the graph of $y = 2\cos x$ as x varies from 0° to 180° inclusive at intervals of 30°. [4]

b Using the same set of axes as in *a*, draw the graph of $y = \cos 2x$ as x varies from 0° to 180° inclusive at intervals of 30°. [4]

c From the graphs determine how many values of x from 0° to 180° inclusive there are for which $2\cos x = \cos 2x$. [2]

*25 Given $R = \cos 120° + i \sin 120°$ and $S = \cos 240° + i \sin 240°$; indicate whether *each* of the following statements is true or false: [10]

a $S = R^2$

b $R = S^2$

c $R \times S = 1$

d $R + S = 1$

e $\sqrt{R} = \cos 60° + i \sin 60°$

Part III

Answer at least two questions from this part.

26 Two roads intersect at an angle of 65° 20′. A triangular corner lot has a frontage of 125 feet on one road and 168 feet on the other road. Find the area of the lot correct to the *nearest square foot*. [10]

27 A ship S, anchored in New York harbor, is sighted from two observations point, A and B, which are 2160 feet apart. In triangle ABS, if angle $A = 18°\ 10'$ and angle $B = 85°\ 40'$, find, correct to the *nearest foot*, the distance between the ship and the nearer observation point. [10]

28 In triangle ABC, $A = 52°\ 20'$, $C = 67°\ 40'$ and $AC = 83$ inches. Find, correct to the *nearest inch*, the length of the altitude on AC. [10]

29 A tunnel is to be built in a straight line between two points, A and B, on opposite sides of a hill. At a point C, 910 feet from A and 720 feet from B, the angle subtended by the line of the tunnel is 74° 20′. Find, correct to the *nearest foot*, the length of the proposed tunnel. [10]

* This question is based on one of the optional topics in the syllabus.

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