The University of the State of New York

280TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, January 23, 1941 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II and III

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two parts, including at least *two* questions from each part.

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Fill in the following lines:

Name of school.................................... Name of pupil....................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express 120° in radians. 1...............

2 Express sin 160° as a function of a positive acute angle. 2...............

3 What is the numerical value of cos (—300°)? 3...............

4 Find the logarithm of 2073. 4...............

5 Find log sin 41° 24′. 5...............

6 Find cos 22° 14′. 6...............

7 If log tan $x = 9.4631 - 10$ and x is a positive acute angle, find the value of x correct to the *nearest minute*. 7...............

8 Find the positive value of $\sin(\tan^{-1} 1)$. [Answer may be left in radical form.] 8...............

9 Express $\tan^2 A$ as a function of $\cos A$. 9...............

10 Complete the formula $\cos(x + y) =$ 10...............

11 If $\tan x = \frac{1}{2}$, express as a common fraction the value of $\tan 2x$. 11...............

12 Find the positive acute angle that satisfies the equation $4 \sin^2 x - 1 = 0$. 12...............

13 If two adjacent sides of a triangle are 5 and 8 and the included angle is 30°, find the area of the triangle. 13...............

14 In triangle ABC, $\sin A = \frac{1}{4}$, $a = 6$, $b = 20$; find $\sin B$. 14...............

15 In triangle ABC, $b = 5$, $c = 4$, $\cos A = \frac{1}{8}$; find a. 15...............

16 In triangle ABC, $b = 8$, $c = 6$, $(B + C) = 120°$; find $\tan \frac{1}{2}(B - C)$. [Answer may be left in radical form.] 16...............

Directions (questions 17–20) — Indicate whether each statement is true or false by writing the word *true* or *false* on the dotted line at the right.

17 An angle whose secant is negative and whose cosecant is positive lies in the third quadrant. 17...............

18 As an angle increases from 180° to 270°, its tangent increases from 0 to ∞. 18...............

19 The maximum value of $\sin 2A$ is 2. 19...............

20 Two different triangles can be formed in which $A = 27°$, $a = 20$ and $b = 22$. 20...............

See instructions for parts II and III on page 1.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from this part.

21 *a* Prove the identity: $\dfrac{\sin (x + y)}{\sin (x - y)} = \dfrac{\tan x + \tan y}{\tan x - \tan y}$ [5]

b Solve the equation $\cos 2x - \sin x = 0$ for all values of x between 0° and 360°. [5]

22 Derive the law of cosines for the acute triangle. [10]

23 Derive the formula $\sin A + \sin B = 2 \sin \dfrac{A + B}{2} \cos \dfrac{A - B}{2}$ [10]

24 Draw the graph of $y = 2 \sin x$ as x varies from 0° to 180° inclusive at intervals of 30°. [10]

*25 Using De Moivre's Theorem, solve completely the equation $x^3 - 1 = 0$ [10]

Part III

Answer at least two questions from this part.

26 In triangle ABC, $BC = 76$ feet, $A = 51° 20'$, $B = 69° 40'$; find the length of AB correct to the *nearest foot.* [10]

27 Find, correct to the *nearest minute,* the angle subtended by an object 72 feet long, if the eye of the observer is 57 feet from one end of the object and 81 feet from the other. [10]

28 From two points on level ground in line with the foot of a tree and on the same side of the tree, the angles of elevation of the top of the tree are 41° and 47°. If the distance between the two points is 50 feet, find, correct to the *nearest foot,* the height of the tree. [10]

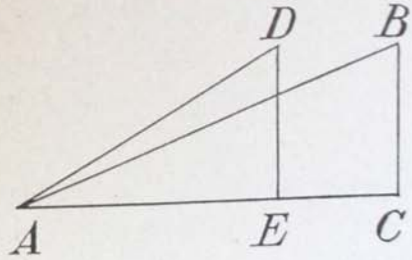
29 In the drawing, DE and BC are each perpendicular to AC and DE equals BC.

a Express $\tan \angle EAD$ in terms of DE and AE. [1]

b Express $\cot \angle CAB$ in terms of AC and BC. [1]

c Show that $AE \tan \angle EAD = \dfrac{AC}{\cot \angle CAB}$. [2]

d Show that $AE (\tan \angle EAD \cot \angle CAB - 1) = EC$. [6]



* This question is based on one of the optional topics in the syllabus.