The University of the State of New York

277TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, January 25, 1940—9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two groups, including at least *two* questions from each group.

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Fill in the following lines:

Name of school..... Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–16) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 The numerical value of $\tan \frac{\pi}{3}$ is .... 1.....

2 The numerical value of $\cos (-300°)$ is .... 2.....

3 The solution of the equation $3 \tan A = 2$, expressed as an inverse function, is $A =$ .... 3.....

4 Expressed to the *nearest tenth*, the number whose logarithm is 2.5604 is .... 4.....

5 If $\sin A = .8930$ and A is less than 90°, the value of A correct to the *nearest minute* is .... 5.....

6 The value of $\log \cos 29° 33'$ is .... 6.....

7 The formula for $\cos^2 \frac{1}{2}A$ in terms of $\cos A$ is $\cos^2 \frac{1}{2}A =$ .... 7.....

8 If A is an acute angle, $\csc A$, expressed as a function of $\cos A$, is $\csc A =$ .... 8.....

9 If x and y are acute angles, and $\sin x = \frac{3}{5}$ and $\cos y = \frac{12}{13}$, the numerical value of $\sin (x - y)$ is .... 9.....

10 The value of x greater than 0° and less than 360° which satisfies the equation $2 \sin^2 x - 3 \sin x = 0$ is .... 10.....

11 In an isosceles triangle the vertex angle is 50° and the length of the base is 30 inches. The length of the altitude drawn upon the base, correct to the *nearest integer*, is ... inches. 11.....

12 In $\triangle ABC$, $c = 20$, $b = 14$, $A = 45°$. The area of $\triangle ABC$ is .... [Answer may be left in radical form.] 12.....

13 In $\triangle ABC$, if $a = 2$, $b = 6$ and $c = 7$, then the numerical value of $\cos B$ is .... 13.....

14 In $\triangle ABC$, if $A = 75°$, $B = 15°$ and $(a + b) = 12$, then $(a - b) =$ .... [Answer may be left in radical form.] 14.....

15 In $\triangle ABC$, if $A = 30°$, $B = 45°$ and $a = 10$, then $b =$ .... [Answer may be left in radical form.] 15.....

16 If $\tan x = \frac{1}{2}$, the value of $\tan 2x$ is .... 16.....

Directions (questions 17–20) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a, b or c.

17 Sin 40° + sin 20° equals (a) sin 60°, (b) cos 20° or (c) cos 10°. 17.....

18 If $\tan A = x$, then $\cot (180° - A)$ equals (a) $\frac{1}{x}$, (b) $-\frac{1}{x}$ or (c) $-x$. 18.....

19 If $\log b = x$, then $\log 100b$ equals (a) $100x$, (b) $2x$ or (c) $x + 2$. 19.....

20 The maximum value of $\sin 2x + \cos y$ is (a) 1, (b) 2 or (c) 3. 20.....

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See instructions for groups II and III on page 1.

Answer five questions from groups II and III, including at least two questions from each group.

Group II

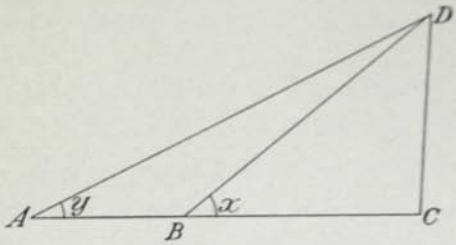
Answer at least two questions from this group.

21 *a* Prove the identity: $\tan y = \dfrac{\cos(x-y)}{\sin x \cos y} - \cot x$ [4]

b Solve the following equation for all values of x greater than $0°$ and less than $360°$: $\csc x - 2\sin x = \cot x$ [6]

22 Starting with the formula for the cosine of the sum of two angles, derive the formula for the sine of half an angle. [10]

23 Given right triangle ADC, B any point on AC and line BD drawn. Derive a formula for DC in terms of AB, angle x and angle y. [10]



24 *a* Draw the graph of $y = \sin x$ as x varies from $0°$ to $180°$ inclusive in intervals of $30°$. [3]

b Using the same set of axes as in *a*, draw the graph of $y = \cos 2x$ as x varies from $0°$ to $180°$ inclusive in intervals of $15°$. [6]

c How many values of x from $0°$ to $180°$ inclusive are there for which $\sin x = \cos 2x$? [1]

*25 Prove:

$$r(\cos\theta + i\sin\theta) \times r'(\cos\phi + i\sin\phi) = rr'[\cos(\theta+\phi) + i\sin(\theta+\phi)]$$ [10]

Group III

Answer at least two questions from this group.

26 In $\triangle ABC$, $AB = 81$ feet, $A = 61°$, $C = 73°$; find the length of AC correct to the *nearest foot*. [10]

27 From a point on level ground the angle of elevation of the top of a hill is $14°\ 10'$. From a second point 1000 feet nearer the foot of the hill the angle of elevation of its top is $17°\ 50'$. Find the height of the hill correct to the *nearest foot*. [10]

28 From a point 175 feet from one end of a wall and 264 feet from the other end the wall subtends an angle of $50°$. Find, correct to the *nearest foot*, the length of the wall. [10]

29 A vertical tower stands at the top of a hill which is inclined $16°$ to the horizontal. At a point 95 feet down the hill from the base of the tower, the tower subtends an angle of $38°$. Find, correct to the *nearest foot*, the height of the tower. [10]

* This question is based on one of the optional topics in the syllabus.