The University of the State of New York

274TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Tuesday, January 24, 1939 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two groups, including at least *two* questions from each group.

Fill in the following lines:

Name of school..... Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–5) — Write the number that represents the value of each.

1 Tan $(-135°)$ 1.....

2 Cos π 2.....

3 Sin $(\csc^{-1} 2)$ 3.....

4 Log sin 42° 42′ 4.....

5 Log 0.0342 5.....

6 Find, correct to the *nearest minute*, the positive acute angle A for which $\tan A = 0.4322$ 6.....

7 If $\sin A = -\frac{4}{5}$ and A is in the fourth quadrant, what is the value of $\sin 2A$? 7.....

8 Find the area of triangle ABC if $a = 6$, $b = 13$ and $C = 30°$ 8.....

9 If in triangle ABC, $a = 8$, $b = 6$ and $c = 5$, find $\cos B$. 9.....

10 If in triangle ABC, $A = 30°$, $B = 105°$ and $a = 10$, find c. [Answer may be left in radical form.] 10.....

11 Express $\cot^2 x$ in terms of $\sin x$. 11.....

12 The radius of a circle is 10 inches. A chord of the circle subtends an angle of 40° at the center. Find, correct to the *nearest inch*, the distance of the chord from the center of the circle. 12.....

13 Express $\tan^2 \frac{x}{2}$ in terms of $\cos x$. 13.....

14 Find the value of $\sin (A + B)$, if $\sin A = \frac{3}{5}$, $\sin B = \frac{4}{5}$ and A and B are acute. 14.....

15 Which of the following statements is never true: (a) $\sin 2A < 2 \sin A$, (b) $\sin 2A = 2 \sin A$, (c) $\sin 2A > 2 \sin A$? 15.....

16 For how many values of x between 0° and 180° do the curves $y = \cos x$ and $y = \tan x$ intersect? 16.....

17 Express the value of $\frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)}$ in terms of a and b. 17.....

18 As an angle increases from 90° to 180°, its cosecant (a) decreases from 1 to -1, (b) increases from $-\infty$ to -1 or (c) increases from 1 to ∞. Which is correct, (a), (b) or (c)? 18.....

19 Find the value of x between 90° and 180° for which $2 \sin^2 x - 1 = 0$ 19.....

20 How many different triangles may be formed in which $a = 7$, $b = 6$ and $B = 30°$? 20.....

224

See instructions for groups II and III on page 1.

Answer <u>five</u> questions from groups II and III, including at least two questions from each group.

Group II

Answer at least two questions from this group.

21 *a* Find, correct to the *nearest degree*, the positive acute angle x that satisfies the equation $3 \cos^2 x + 8 \sin x - 7 = 0$ [6]

b Prove the identity: $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$ [4]

22 Assuming the formulas for $\sin (A + B)$ and $\cos (A + B)$, derive the formula for $\tan 2A$ in terms of $\tan A$. [10]

23 In right triangle ABC, C is the vertex of the right angle and D is any point in CB. Show that

a $DB = AC \ (\tan CAB - \tan CAD)$ [6]

b $DB = \dfrac{AC \sin (\angle CAB - \angle CAD)}{\cos CAB \cos CAD}$ [4]

24 *a* Draw the graph of the equation $y = \tan x$ as x varies from 0° to 90° in intervals of 15°. [7]

b Using the same set of axes as in *a*, plot the graph of the equation $y = 3$ [1]

c With the aid of the graphs made in answer to *a* and *b*, find, correct to the *nearest degree*, a positive acute angle x which is an approximate solution of the equation $\tan x = 3$ [2]

*25 Express in polar form:

a The product of $1 + i\sqrt{3}$ and $1 - i$ [7]

b The real root of $x^3 - 1 = 0$ [3]

Group III

Answer at least two questions from this group.

26 The captain of a ship which is sailing north at the rate of 15 miles per hour observes a lighthouse N 35° W. One hour later he observes this lighthouse to be S 70° W. Find, correct to the *nearest mile*, the distance of the ship from the lighthouse at the second observation. [10]

27 The top T of a tower is sighted by two observers, O_1 and O_2, who are on the same side of the tower, 600 feet apart, and in the same vertical plane with T. The angles of elevation of T from O_1 and O_2 are 28° and 74° respectively. Find, correct to the *nearest foot*, the height of the tower. [10]

28 The distances from an observer's eye to the summits of two hills are 10 miles and 28 miles. If the angle between his lines of sight is 104°, find, correct to the *nearest mile*, the shortest distance from one summit to the other. [10]

29 A body is acted upon by two forces of 146 pounds and 206 pounds. If their resultant is 300 pounds, find, correct to the *nearest minute* the angle at which the resultant is inclined to the force of 146 pounds. [10]

* This question is based on one of the optional topics in the syllabus.