The University of the State of New York

271ST HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, January 27, 1938 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two groups, including at least *two* questions from each group.

See instructions for groups II and III on page 1.

Answer five questions from groups II and III, including at least two questions from each group.

Group II

Answer at least two questions from this group.

21 *a* Solve for x the equation $9 \cos^2 x = 10 - 6 \sin x$, expressing the angle as an inverse function. [6]

b Prove the identity: $\dfrac{1 + \sec^2 A}{\tan A} = \dfrac{1 + \cos^2 A}{\sin A \cos A}$ [4]

22 Derive the formula for $\cos (x + y)$ in which x, y and $(x + y)$ are positive acute angles. [10]

23 *a* Plot the graph of the equation $y = \sin 2x$, as x varies from 0° to 180° inclusive in intervals of 15°. [6]

b Using the same set of axes as in *a*, plot the graph of the equation $y = 2 \sin x$, as x varies from 0° to 180° inclusive in intervals of 30°. [4]

24 *a* Represent the *six* functions of an angle in the first quadrant as line segments. Give a full description of your diagram. [4]

b For each function indicate the line segment representing it. [6]

*25 *a* Using the relationships between rectangular and polar coordinates, show that the equations $y^2 + x^2 - 10x = 0$ (in rectangular coordinates) and $r = 10 \cos \theta$ (in polar coordinates) represent the same curve. [3]

b Sketch the graph of the curve indicated in *a*, using the following values for θ: 0°, ± 30°, ± 45°, ± 60°, ± 90°. [7]

Group III

Answer at least two questions from this group.

26 The diagonals of a parallelogram are 96 and 72 and the shorter side is 30; find, correct to the *nearest minute,* the acute angle formed by the two diagonals. [5, 5]

27 In triangle ABC, $a = 35$, $b = 42$, $C = 75° 28'$; find A. [10]

28 A tree stands at the end of a straight path which is inclined 12° to the horizontal. From a point down the path 100 feet from the foot of the tree the angle subtended by the tree is 20°; find, correct to the *nearest foot,* the height of the tree. [4, 6]

29 Two land stations, A and B, equipped with radio beacons, are 150 miles apart. B is due north of A. A ship in a dense fog communicates with both stations in order to determine its position. If the ship is N. 65° 30′ E. from A and S. 34° 30′ E. from B, find, correct to the *nearest mile,* how far it is from A. [4, 6]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express cot x in terms of sin x and cos x. 1.....

2 Write the formula for tan $2x$ in terms of tan x. 2.....

3 Find the number of degrees in the acute angle A if cos A = sin 23° 3.....

4 Express 225° in radian measure. [Answer may be left in terms of π.] 4.....

5 Find, correct to *four decimal places*, the number whose logarithm is 9.7523 — 10 5.....

6 Find log sin 37° 24′ 6.....

7 Find, correct to the *nearest minute*, the positive acute angle A for which cos A = 0.9318 7.....

8 Write the positive value of sin $(\tan^{-1} \frac{5}{12})$. 8.....

9 Write the numerical value of cos (—300°). 9.....

10 If log n = log a — log b, express n in terms of a and b. 10.....

11 In triangle ABC, $a = 10$, sin $A = .30$, sin $C = .24$; find c. 11.....

12 In triangle ABC, $a = 5$, $b = 10$, $c = 12$; find cos C. 12.....

13 In triangle ABC, $a + b = 10$, $a - b = 7$, $A - B = 90°$; find the value of tan $\frac{1}{2}(A + B)$. 13.....

14 If cos $A = -\frac{7}{25}$, find, without the use of tables, the positive value of sin $\frac{1}{2}A$. 14.....

15 Sin $(30° + x)$ is equal to (a) $\frac{1}{2}$ + sin x, (b) $\frac{\cos x + \sqrt{3}\sin x}{2}$ or (c) $\frac{\cos x - \sqrt{3}\sin x}{2}$; which is correct, (a), (b) or (c)? 15.....

16 Find the value of A between 180° and 270° that satisfies the equation $3\tan^2 A = 1$ 16.....

17 Two sides of a parallelogram are 24 inches and 10 inches and the area is 120 square inches. Find an acute angle of the parallelogram. 17.....

18 For what value of x between 0° and 180° are the ordinates of the curves y = sin x and y = cos x equal? 18.....

19 As A increases from 180° to 270°, tan A increases (a) from 0 to + ∞, (b) from — ∞ to + ∞ or (c) from — ∞ to 0; which is correct, (a), (b) or (c)? 19.....

20 The maximum value of $\frac{1}{2}$ sin $2x$ is (a) greater than, (b) equal to or (c) less than, the maximum value of 2 sin $\frac{1}{2}x$; which is correct, (a), (b) or (c)? 20.....