The University of the State of New York

268TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, January 21, 1937 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two groups, including at least *two* questions from each group.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express sin 118° as a function of a positive acute angle less than 45°. *Ans*.....

2 Write the numerical value of sin (—210°) *Ans*.....

3 Express 135° in radian measure. [Answer may be left in terms of π.] *Ans*.....

4 Write the positive value of cos $(\cot^{-1} \frac{3}{4})$ *Ans*.....

5 Complete the formula: sin $(A - B) = \ldots$ *Ans*.....

6 Find the four-place decimal whose logarithm is 9.4260—10 *Ans*.....

7 Find, correct to the *nearest minute*, the positive acute angle whose cosine is .5633 *Ans*.....

8 Find log sin 62° 23′ *Ans*.....

9 Given log tan $A = 9.8971—10$; find, correct to the *nearest minute*, the value of A if A is a positive acute angle. *Ans*.....

10 From a ship's masthead 175 feet high, the angle of depression of a boat is observed to be 28° 42′; find, correct to the *nearest foot*, the distance of the boat from the ship. *Ans*.....

11 What is the maximum value of $2 \sin 3x$? *Ans*.....

12 Two sides of a parallelogram are 20 and 12 and their included angle is 60°: find the area of the parallelogram. [Answer may be left in radical form.] *Ans*.....

13 In triangle ABC, $a = 10$, $c = 12$, $\sin C = .4$; find $\sin A$. *Ans*.....

14 In triangle ABC, $b = 8$, $c = 5$, $A = 60°$; find a. *Ans*.....

15 In triangle ABC, $A = 75°$, $B = 15°$; find the numerical value of the ratio $\frac{a+b}{a-b}$ [Answer may be left in radical form.] *Ans*.....

16 Find the value of A in the *third* quadrant that satisfies the equation $2 \sin^2 A = 1$ *Ans*.....

17 If $\cos x = .02$, find, without the use of tables, the value of $\sin \frac{1}{2} x$ *Ans*.....

18 Given a, b and A of triangle ABC with b less than a; then B may have (a) two solutions, (b) one solution or (c) no solution. Which is correct, (a), (b) or (c)? *Ans*.....

19 As the cosine increases from —1 to 0, the cotangent (a) increases from 0 to ∞, (b) decreases from ∞ to 0 or (c) decreases from 0 to —1. Which is correct, (a), (b) or (c)? *Ans*.....

20 If $\tan A = S$ and $\cot (180° - A) = R$, then (a) $R = S$, (b) $R = \frac{1}{S}$ or (c) $R = -\frac{1}{S}$. Which is correct, (a), (b) or (c)? *Ans*.....

2.16

See instructions for groups II and III on page 1.

Answer five questions from groups II and III, including at least two questions from each group.

Group II

Answer at least two questions from this group.

21 *a* Starting with the law of sines, derive the law of tangents. [7]

b Starting with the formula for $\cos 2A$, derive the formula for $\sin \frac{1}{2}A$. [3]

22 *a* Solve the following equation for all values of A from $0°$ to $360°$:

$$\cos^2 A - 3 \sin A + 3 \sin^2 A = 0 \quad [6]$$

b Prove the identity: $\tan x = \dfrac{\sin 2x}{1 + \cos 2x}$ [4]

23 *a* Using the same set of axes, plot the graph of $y = \sin x$ and the graph of $y = 2 \cos x$, as x varies from $0°$ to $180°$ in intervals of $30°$. [3, 5]

b From the graphs made in answer to *a* estimate the value of x common to both equations. [2]

*24 Given the equation $x^4 + 1 = 0$

a Using DeMoivre's Theorem, express the roots in polar form. [8]

b Express the roots in radical form. [2]

Group III

Answer at least two questions from this group.

25 In triangle ABC, $AB = 19$, $BC = 34$, $AC = 49$; find angle C. [2, 8]

26 A straight road leads up a hill from a bank of a river. The road is at right angles to this bank and is inclined at an angle of $16°$ to the horizontal. At a point on this road 360 feet from the river the angle of depression of a point directly opposite on the other bank of the river is $6° 30'$. Find the width of the river correct to the *nearest foot*. [6, 4]

27 A bridge represented by line AB crosses a ravine. A vertical cross section through AB cuts the sides of the ravine in lines AC and BC. If the bridge is 125 feet long and the sides of the ravine are inclined to the horizontal at angles of $60° 25'$ and $51° 20'$, find the height of the bridge above point C. [5, 5]

28 Two forces, one of 400 pounds and the other of 600 pounds, act on a body and make an angle of $52° 20'$ with each other. Find the direction of the resultant force and the magnitude of this force correct to the *nearest pound*. [3, 7]

* This question is based on one of the optional topics in the syllabus.