The University of the State of New York

228TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Wednesday, January 19, 1927 — 1.15 to 4.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement for plane trigonometry is five recitations a week for half a school year or the equivalent.

Answer seven questions, including three from group I and four from group II.

A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

In the examination in plane trigonometry the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

Group I

Answer three questions from this group.

1 A light on a certain steamer is known to be 40.5 feet above the water. An observer on the shore, whose instrument is 5.6 feet above the water, finds the angle of elevation of this light to be 4° 20′. What is the distance from the observer to the steamer? [10]

2 The distances of a station from two objects situated on opposite sides of a hill are 1076 yards and 988 yards respectively and the angle subtended at the station by the line joining the two objects is 62° 34′. What is the distance between the two objects? [10]

3 *a* A side of an equilateral triangle is 12. One side is trisected and the points of division joined to the opposite vertex. Find the number of degrees in each of the parts into which the angle at this vertex is divided. [14]

b Is the answer to *a* the same for all equilateral triangles? [2]

4 A flagstaff *k* feet high stands on top of a cliff. An observer in the same horizontal plane with the base of the cliff finds the angles of elevation of the top and bottom of the flagstaff to be *A* and *B* respectively. Find the height of the cliff in terms of *A*, *B* and *k*. [10]

Group II

Answer four questions from this group.

5 *a* Complete the Law of Cosines in the form beginning $a^2 = \ldots\ldots$ Prove this law if *A* is acute; if *A* is obtuse. [1, 4, 5]

b If *x* is an angle in the first quadrant, prove by means of a diagram that $\sin(180° - x) = \sin x$. [5]

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6 *a* From the equation $3 \sin 2x = (\sin x + \cos x)^2$ find the value of $\sin 2x$. [7]

b Solve for all values of *x* between 0° and 360°:

$$2 \sin 2x = \sqrt{3} \quad [6]$$

7 *a* If $\cos 300° = a$ prove that $\cot 120° = -\frac{a}{\sqrt{1-a^2}}$ [8]

b If $\sin x - 3 \cos x = 0$, find the value of $\tan x$ without first finding the value of *x*. [5]

8 Given a right isosceles triangle with a median drawn to one of the equal sides. Show that this median divides the angle at the vertex from which it is drawn into two parts whose tangents are $\frac{1}{3}$ and $\frac{1}{2}$. [2, 11]

9 *a* Draw the graph of $y = 2 \sin x$, taking values of *x* from 0° to 360° at intervals of 30°. [9]

b From the graph made in answer to *a* determine the values of *x* which give the greatest and smallest values of *y*. [2]

c Use the graph made in answer to *a* to solve for *x* the equation $2 \sin x = \frac{1}{2}$. [2]

10 *a* Prove that $\frac{\sin(A+B)}{\sin(A-B)} = \frac{\tan A + \tan B}{\tan A - \tan B}$ [7]

b Prove that $1 + \cos A = 2 \cos^2 \frac{A}{2}$ [6]

11 *a* Complete each of the following:

(1) If $\sin x = -\cos x$, then *x* must be an angle in the ..... or ..... quadrant. [4]

(2) From the formula $K = \frac{1}{2} bc \sin A$ for the area of a triangle, it is evident that the area *K* is the greatest possible for any given values of *b* and *c*, when *A* contains ..... degrees. [3]

b State whether each of the following is true or false: [Label each answer with the corresponding letter.] [6]

(1) $\sin(x - 180°) = \sin x$

(2) $\cos(x - 180°) = \cos x$

(3) $\tan(x - 180°) = -\tan x$

(4) $\cot(x - 180°) = -\cot x$