The University of the State of New York

226TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Wednesday, January 18, 1922—1.15 to 4.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement for plane trigonometry is two recitations a week for a school year.

Answer seven questions, including two from group I, two from group II and three from group III.

A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

In the examination in plane trigonometry the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

Group I

Answer two questions from this group.

1 Prove that $\cos (x + y) = \cos x \cos y - \sin x \sin y$ when x, y and $(x + y)$ are each less than $90°$. [13]

2 *a* With the aid of a drawing derive the $\sin (90° + x)$ and $\cos (90° + x)$ in terms of functions of x when x is any positive acute angle. [7]

b Trace the changes in $\cos x$ as x varies from $0°$ to $360°$. [6]

3 Prove that $\cos U - \cos V = -2 \sin \dfrac{U+V}{2} \sin \dfrac{U-V}{2}$ [13]

Group II

Answer two questions from this group.

4 Prove the following identities:

a $(\csc A - \cot A)^2 = \dfrac{1 - \cos A}{1 + \cos A}$ [6]

b $\dfrac{\sin 2A}{1 + \cos 2A} = \tan A.$ [7]

5 *a* Solve for all values of x from $0°$ to $360°$ inclusive:
$\cos 2x + \sin x = 1$ [7]

b Determine an angle x in the second quadrant satisfying the equation
$\tan (45° - x) + \tan (45° + x) = 4$ [6]

6 *a* Evaluate (without the use of tables):
$$\frac{\cos 60° + \cot 225° + \sin 270°}{\sec (-60°) - \tan 180° + \sin 210°}$$ [7]

b Express $\tan x$ and $\sec x$ in terms of $\sin x$. [6]

Group III

Answer three questions from this group.

7 A tree is broken by the wind; its top strikes the ground 32 feet from the foot of the tree and makes an angle of $35° 54'$ with the ground. Find the original height of the tree. [16]

8 The sides of a triangle are 18.723, 28.14 and 35.817; find the length of the perpendicular from the largest angle upon the opposite side. [16]

9 Two forces, 125.0 lb and F lb, include an angle of $72° 15'$ between their directions; if their resultant force makes an angle of $31° 8'$ with the 125.0 lb force, find F. [The resultant is represented by the longer diagonal of the parallelogram whose sides are 125.0 and F, the included angle being $72° 15'$.] [16]

10 At two stations on the same horizontal plane the height of a kite subtends the same angle A. The angle which the line joining one station and the kite subtends at the other station is B; the distance between the two stations is a. Show that the height of the kite is $\frac{1}{2} a \sin A \sec B$. [16]