The University of the State of New York

EXAMINATION FOR QUALIFYING CERTIFICATES 217

PLANE GEOMETRY

Tuesday, September 14, 1926 — 9.15 a. m. to 12.15 p. m., only

Answer eight questions. Irrational results may be left in the form of π and radicals unless otherwise stated. Papers entitled to less than 75 credits will not be accepted.

1 Prove that the diagonals of a parallelogram bisect each other.

2 Prove that an angle formed by two chords intersecting each other within a circle is measured by one half the sum of the arc intercepted between its sides and the arc intercepted between the sides of its vertical angle.

3 Prove that the bisector of an angle of a triangle divides the opposite side into segments proportional to the adjacent sides.

4 *a* Construct the mean proportional between two given straight lines.

b Construct the locus of all points within a circle that are equidistant from the ends of a given chord.

5 Using a given line as an altitude, construct a triangle similar to a given triangle.

6 The area of a circle is 201$\frac{1}{7}$ square inches; find the area of an inscribed equilateral triangle. [Use $\pi = 3\frac{1}{7}$]

7 Prove that the product of any side of a triangle and the corresponding altitude is equal to the product of any other side and the corresponding altitude.

8 ABC is an isosceles triangle inscribed in a circle; the vertex angle B is 30° and D is the middle point of the arc BC. If the line AD intersects the chord BC at E, how many degrees are there in the angles DEC and BED?

9 In an isosceles trapezoid each of the nonparallel sides is 8, the shorter base is 12 and the angles at the ends of the larger base are 60° each. In an equivalent rhombus one of the diagonals is 8. Find the other diagonal.

10 In an isosceles triangle ABC, in which CA equals CB, AB is extended through B any distance to D, and D is joined to C. Prove that angle DAC is greater than angle D.

Group III

Irrational results may be left in the form of π and radicals unless otherwise stated.

7 A triangle that circumscribes a given circle has two of its angles 44° and 36°; find the three angles of the inscribed triangle formed by joining the points of tangency of the sides of the given triangle. [12$\frac{1}{2}$]

8 ABC is a triangle in which side AB is greater than side AC. OB and OC bisect angles B and C respectively. Show that OB is greater than OC. [12$\frac{1}{2}$]

9 *a* Prove that if two sides of a triangle are equal, the medians to these two sides are equal. [9]

b State the converse of *a*. [Proof not required in *b*] [3$\frac{1}{2}$]

10 $ABCDEF$ is a regular hexagon inscribed in a circle whose radius is 8; ACE is the triangle formed by joining alternate vertices of the hexagon.

a Find the area of the hexagon $ABCDEF$. [4]

b Find the area of the triangle ACE. [8$\frac{1}{2}$]

11 A flywheel 30 inches in diameter is running at 35 revolutions per minute; find the speed in feet per minute of a point on the rim of the flywheel. [12$\frac{1}{2}$]

12 ABC is a triangle with base $AB = 10''$ and altitude $CD = 12''$; EF is a line 8″ long parallel to AB and terminated by the other sides of the triangle.

a Find the area of triangle EFC. [6$\frac{1}{2}$]

b Find the distance of the line EF from base AB. [6]

13 The radii of two circles are 8 and 3 and the length of the common external tangent is 12; find (*a*) the distance between the centers of the two circles [6], (*b*) the distance from the center of the smaller circle to the point of intersection of the line through the centers and the external tangent extended to meet [6$\frac{1}{2}$].