The University of the State of New York

EXAMINATION FOR QUALIFYING CERTIFICATES

PLANE GEOMETRY

Tuesday, September 16, 1924 — 9.15 a. m. to 12.15 p. m., only

Answer eight questions. Irrational results may be left in the form of π and radicals unless otherwise stated. Papers entitled to less than 75 credits will not be accepted.

1 Prove that if two right triangles have the hypotenuse and a leg of one respectively equal to the hypotenuse and a leg of the other, the triangles are congruent.

2 Prove that an angle formed by two secants intersecting without a circle is measured by one half the difference of the intercepted arcs.

3 Prove that the area of a triangle is equal to half the product of its base and its altitude.

4 *a* Given a fixed line *AB*. On *AB* as the hypotenuse right triangles are constructed. What is the locus of the vertices of the right angles?

  b What is the locus of the centers of the circles that are always tangent to both sides of a given angle *ABC*?

  [No proof required in either *a* or *b*.]

5 The median to the base of a scalene triangle is produced through the base, and perpendiculars are drawn to this line from the extremities of the base. Prove that these perpendiculars are equal.

6 Given one leg of a right triangle and the altitude on the hypotenuse; construct the triangle. [Leave all construction lines on the paper.]

7 If two diagonals of a regular pentagon intersect, prove that the longer segment of each is equal to a side of the pentagon.

8 Find the area of a circle inscribed in an equilateral triangle whose side is 10 inches.

9 The bisector of the vertex angle *A* of the triangle *ABC* intersects the base at *D* and the circumference of the circumscribed circle at *E*. Prove that $AB \times AC = AD \times AE$.

10 The upper (shorter) base of an isosceles trapezoid is 8 inches; each leg is 5 inches and the projection of each leg on the base is 4 inches. Find the side of a square equivalent to the trapezoid.