PLANE GEOMETRY

Tuesday, August 23, 1955 — 8.30 to 11.30 a.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in plane geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1955 or number and length in minutes of lessons taken in the summer of 1955 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (d) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1955 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer three questions from this part.

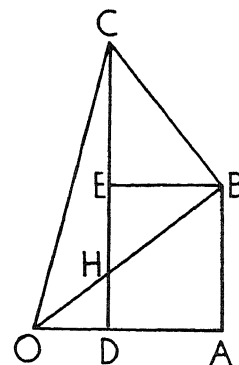26 Prove: The diagonals of a parallelogram bisect each other.   [10]

27 In the accompanying figure, angles OAB, OBC and ODC are right angles. A line through B parallel to AO intersects DC at E.

a Prove that
  (1) angles AOB and ECB are equal   [5]
  (2) triangles AOB and ECB are similar   [2]
b If $AB = 15$ and the ratio of EB to EC is $3:4$, find OA.   [3]

28 Prove: The area of a trapezoid is equal to one half the product of its altitude and the sum of its bases.   [10]

29 Two circles O and O' are tangent internally at A, and the smaller circle passes through the center of the larger circle as shown. AB is a chord of the larger circle and is cut by the smaller circle at C.

a Prove that C is the midpoint of AB.   [8]
b What is the locus of the midpoints of all chords of circle O drawn from point A?   [2]
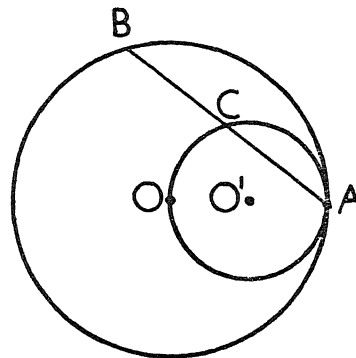
30 In a given circle, M is the midpoint of arc AB. Prove that chord AB is less than twice chord AM.   [10]

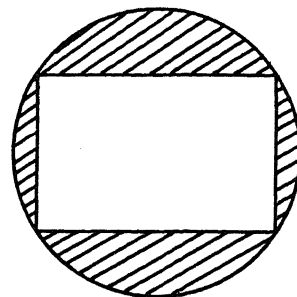[1]   [OVER]

Part III

Answer two questions from this part. Show all work.

31 An ornamental design has the form of a rectangle inscribed in a circle as shown. One side of the rectangle is 12 inches, and the area of the rectangle is 192 square inches. Find the area of the shaded portion of the design. [Use $\pi = 3.14$.]   [10]

32 A quadrilateral is inscribed in a circle as shown. Diagonals *AC* and *BD* intersect at *E*. The tangent at *D* intersects *CA* extended at *F*. Arcs *AB, BC, CD* and *DA* are in the ratio $5:4:2:1$. Find the number of degrees in arc *AD*, angle *AFD*, angle *AEB,* angle *FDC* and angle *FAB*.   [10]
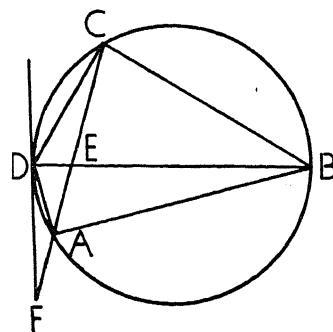
33 A regular pentagon is circumscribed about a circle whose radius is 10 inches.
  a Find to the *nearest inch* the perimeter of the pentagon.   [7]
  b Using the result found in answer to part *a*, find to the *nearest square inch* the area of the pentagon.   [3]

34 The altitude *h* on the hypotenuse of a right triangle divides the hypotenuse into segments *r* and *s*.
  a Express *h* in terms of *r* and *s*.   [2]
  b Express the area *K* of the triangle in terms of *r* and *s*.   [4]
  c Find *K* to the *nearest integer* if $r = 8$ and $s = 12$.   [4]

[2]

PLANE GEOMETRY

Fill in the following lines:

Name of pupil................................Name of school................................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 A base angle of an isosceles triangle is 4 times its vertex angle. Find the number of degrees in the vertex angle.

1................

2 The sum of the interior angles of a polygon is 720°. Find the number of sides of the polygon.

2................

3 The line segment joining the midpoints of two sides of a triangle is 8. Find the third side of the triangle.

3................

4 Find the circumference of a circle whose radius is 21. [Use $\pi = \frac{22}{7}$.]

4................

5 Triangle ABC is inscribed in a circle. If arc $AC = 50°$, find the number of degrees in angle ABC.

5................

6 PAB and PCD are secants of a circle drawn from the external point P. If arc $BD = 80°$ and arc $AC = 20°$, find the number of degrees in angle BPD.

6................

7 A line parallel to side AC of triangle ABC intersects AB at D and BC at E. If $BA = 16$, $BD = 4$ and $BE = 5$, find BC.

7................

8 Corresponding altitudes of two similar triangles are in the ratio $3:2$. If a side of the larger triangle is 15, find the corresponding side of the smaller triangle.

8................

9 The areas of two similar polygons are in the ratio $9:25$. If a side of the smaller polygon is 6, find the corresponding side of the larger polygon.

9................

10 In triangle ABC the altitude from C intersects AB at D. If angle $C = 90°$, $AB = 16$ and $AD = 4$, find AC.

10................

11 Find the area of an equilateral triangle whose side is 10. [Answer may be left in radical form.]

11................

12 In triangle ABC angle $C = 90°$, angle $A = 77°$ and $AB = 40$. Find AC.

12................

Directions (13–19): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a, b* or *c*.

13 If in triangle ABC angle $A = 62°$ and angle $B = 58°$, the shortest side of the triangle is (a) AB (b) AC (c) BC

13................

14 If a side of an equilateral triangle is 20, an altitude of the triangle is (a) $20\sqrt{3}$ (b) $10\sqrt{3}$ (c) $\dfrac{20}{\sqrt{3}}$

14................

15 If a diagonal of a square is 10, a side of the square, correct to the *nearest integer,* is (a) 7 (b) 8 (c) 9

15................

[3]

[OVER]

16 If two circles have only two common tangents, the circles   (a) are tangent internally   (b) intersect   (c) are tangent externally

16................

17 Two chords of a circle intersect within the circle. If the segments of one chord are r and s and the segments of the other are t and v, then

(a) $\dfrac{r}{v} = \dfrac{t}{s}$ (b) $\dfrac{r}{s} = \dfrac{v}{t}$ (c) $\dfrac{r}{v} = \dfrac{s}{t}$

17................

18 Chord AB of a circle is extended through B to point P. If from P a line is drawn tangent to the circle at C and chords CA and CB are drawn, triangle   (a) ACB is similar to triangle CBP (b) ACB is similar to triangle ACP (c) ACP is similar to triangle BCP

18................

19 The number of points which are equidistant from two given intersecting straight lines and which also are at a given distance from their point of intersection is   (a) 1   (b) 2   (c) 4

19................

Directions (20–23): For *each* of the following, tell whether the statement is *always* true, *sometimes* true or *never* true by writing on the line at the right the word *always, sometimes* or *never*.

20 If the diagonals of a parallelogram are equal and perpendicular to each other, the parallelogram is a square.

20................

21 If two parallel lines are cut by a transversal, the exterior angles on the same side of the transversal are supplementary.

21................

22 The altitudes of a triangle intersect inside the triangle.

22................

23 The center of the circle circumscribed about a triangle lies on one side of the triangle.

23................

Directions (24 and 25): Leave all construction lines on your paper.

24 Find by construction the center of the circle which can be inscribed in the triangle ABC.
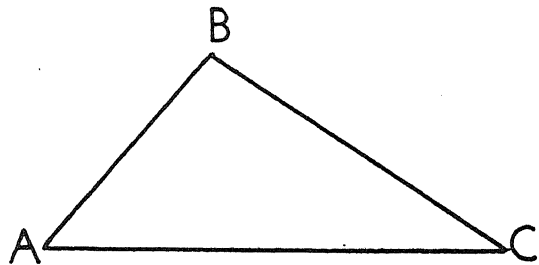
25 Construct an isosceles triangle whose base is b and one of whose legs is a.

[4]
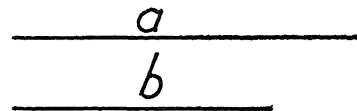
FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
PLANE GEOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 13–19, allow credit if the pupil has written the correct expression instead of the letter *a, b* or *c.*

(1) 20

(2) 6

(3) 16

(4) 132

(5) 25

(6) 30

(7) 20

(8) 10

(9) 10

(10) 8

(11) $25\sqrt{3}$

(12) 9

(13) *b*

(14) *b*

(15) *a*

(16) *b*

(17) *a*

(18) *c*

(19) *c*

(20) always

(21) always

(22) sometimes

(23) sometimes