The University of the State of New York

319TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Wednesday, August 19, 1953 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1953 or number and length in minutes of lessons taken in the summer of 1953 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1953 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

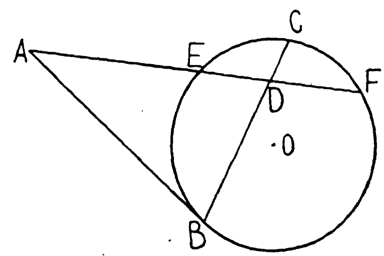
Answer three questions from part II.

26 Prove that the area of a triangle is equal to one half the product of its base and its altitude. [10]

27 Prove that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [10]

28 Given acute angle ABC. Equal line segments BD and BE are laid off on sides BA and BC respectively. A line perpendicular to BA at D intersects BC at S and a line perpendicular to BC at E intersects BA at R. DS intersects ER at O. Prove: $RO = OS$ [10]

29 In the accompanying figure AEF is a secant and AB is a tangent of the circle O. C is the mid-point of arc EF and chord CB meets AF at D. Prove: $AB = AD$ [10]



30 In parallelogram $ABCD$, E is a point on AD and F is a point on DC. The line through E and F meets BC extended at S and BA extended at R. Prove: $RE \times SC = AE \times FS$ [10]

Part III

Answer two questions from part III.

31 In circle O arcs AB, BC, CD and DA are in the ratio $3:4:8:5$. A tangent to the circle at B intersects DA extended at E. Chords AC and BD intersect at F.

a Find the number of degrees in arcs AB, BC, CD and DA. [4]

b Find the number of degrees in angles AFB, DEB and FAE. [6]

32 The radius of the circle circumscribed about a regular polygon of 10 sides is 16.

a Find the apothem to the *nearest integer.* [4]

b Find a side to the *nearest integer.* [4]

c Using the results found in answer to *a* and *b*, find the area of the polygon. [2]

33 The circumference of a circular wading pool is 132 feet.

a Find the radius of the pool. [Use $\pi = \frac{22}{7}$] [2]

b A concrete walk 4 feet wide is built around the pool. Find to the *nearest square foot* the area of the walk. [6]

c Find the cost of the walk at 75¢ per square foot. [2]

34 The bases AB and CD of trapezoid $ABCD$ are 12 and 8 respectively and the area of the trapezoid is 100. Diagonals AC and BD intersect in E.

a Find the altitude of the trapezoid. [2]

b Using similar triangles, find the altitude to AB of triangle AEB. [3]

c Find the area of triangle AEB and of triangle DEC. [3]

d Find the sum of the areas of triangles AED and BEC. [2]

Plane Geometry

Fill in the following lines:

Name of pupil..Name of school..

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The line segment joining the mid-points of two sides of a triangle is 8. Find the third side. 1................

2 The vertex angle of an isosceles triangle is 50°. Find the number of degrees in an exterior angle at the base. 2................

3 Find the sum of the interior angles of a polygon of 9 sides. 3................

4 In a circle, a central angle and an inscribed angle intercept the same arc. If the inscribed angle contains 41°, find the number of degrees in the central angle. 4................

5 A line parallel to side AC of triangle ABC intersects AB at D and BC at E. If $AB = 8$, $BC = 10$ and $AD = 5$, find CE. 5................

6 A tangent and a secant are drawn from an external point to a circle. Find the entire secant if its external segment is 3 and the tangent is 6. 6................

7 Chord AB passes through point P in circle O. If $AP = 6$ and $PB = 4$, find the product of the segments of any other chord through point P. 7................

8 The diagonals of a quadrilateral are 8 and 10 and are perpendicular to each other. Find the area of the quadrilateral. 8................

9 Find the length of an arc of 50° in a circle whose circumference is 72. 9................

10 Corresponding sides of two similar triangles are 15 and 10. If the area of the larger triangle is 45, find the area of the smaller triangle. 10................

11 Find a side of an equilateral triangle whose area is $16\sqrt{3}$ 11................

12 Two sides of a rectangle are 9 and 12. Find a diagonal. 12................

13 Is statement *a* the converse of statement *b*? [Answer *yes* or *no*.]

a The mid-point of the hypotenuse of a right triangle is equidistant from the three vertices.

b A point is equidistant from the three vertices of a right triangle if it is the mid-point of the hypotenuse. 13................

14 The legs of a right triangle are 5 and 8. Find to the *nearest degree* the smaller acute angle. 14................

Directions (15–18) — Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

15 In triangle ABC, if C is a right angle and if CD is the altitude on AB, then (*a*) $AD:DC = DC:AB$ (*b*) $DB:BC = BC:AD$ (*c*) $AD:AC = AC:AB$ 15......

16 The number of points that are equidistant from two given intersecting lines and at a given distance from their point of intersection is (*a*) 1 (*b*) 2 (*c*) 4 16......

17 If two circles intersect, the greatest number of common tangents that can be drawn is (*a*) 0 (*b*) 2 (*c*) 4 17......

18 If the sides of a triangle are unequal, the angle opposite the longest side is (*a*) less than 60° (*b*) equal to 60° (*c*) greater than 60° 18......

Directions (19–23) — In the case of each of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

19 A trapezoid inscribed in a circle is isosceles. 19................

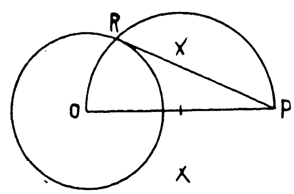
20 If perpendiculars from the center of a circle to the sides of an inscribed polygon are equal, the polygon is equilateral. 20................

21 If two parallel lines are cut by a transversal, the alternate exterior angles are equal. 21................

22 If two parallelograms have equal bases, their areas are to each other as their altitudes. 22................

23 If the bisectors of two given adjacent angles form with each other an angle of 45°, the given angles are complementary. 23................

24 To construct a tangent from point P to circle O, line segment PO is bisected and a semicircle is drawn as shown in the figure. Which one of the following statements is used to prove that PR is a tangent?



a A tangent is a line which touches a circle at one and only one point.

b The line perpendicular to a radius at its extremity on the circle, is tangent to the circle.

c A tangent is perpendicular to a radius at the point of contact. 24................

25 Construct the locus of points equidistant from the given points A and B. [Leave all construction lines on your paper.]

A

B