PLANE GEOMETRY

Wednesday, August 20, 1952 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1952 or number and length in minutes of lessons taken in the summer of 1952 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1952 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer three questions from part II.

26 Prove that if the opposite sides of a quadrilateral are equal, the figure is a parallelogram. [10]

27 Prove that in a circle an angle formed by a tangent and a chord drawn from the point of contact is measured by one half the intercepted arc. [10]

28 *AB* is a diameter of a circle whose center is *O*. From point *P* on the circle a line is drawn perpendicular to *AB* meeting it at *N*, and *OP* is drawn. The bisector of angle *OPN* meets the circle at *R*. If *OR* is drawn, prove that

    a *OR* is parallel to *PN*   [6]

    b arc *AR* = arc *RB*   [4]
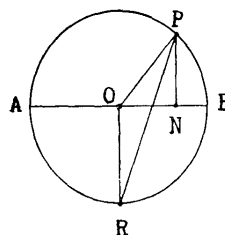
29 Triangles *ABC* and *ADC*, lying on opposite sides of *AC*, are equal in area. Prove that

    a the perpendicular from *B* to *AC* is equal to the perpendicular from *D* to *AC*   [4]

    b *AC* bisects line segment *BD*   [6]

[1]   [OVER]

30 In the case of *each* of the following, tell whether the information given in the hypothesis is (1) too much to justify the conclusion or (2) too little to justify the conclusion.

  a If the diagonals of a quadrilateral are perpendicular to each other, the figure is a rhombus.   [2]

  b The median of a triangle is equal to one half the side to which it is drawn.   [2]

  c The areas of two triangles having equal bases and unequal altitudes are to each other as their altitudes.   [2]

  d If a polygon is equiangular, a circle can be circumscribed about it.   [2]

  e If two circles intersect, the common chord of these circles is the perpendicular bisector of the line segment joining the centers of the circles.   [2]

Part III
### Answer two questions from part III.

31 Chord *AB* is 16 inches long and is located 15 inches from the center *O* of a circle.

  a Find the length of the radius of the circle.   [3]

  b Find angle *AOB* to the *nearest degree*.   [4]

  c Using the results found in answer to *a* and *b*, find the area of minor sector *AOB* to the *nearest square inch*.   [3]

32 The longer base of a trapezoid is 4 more than the shorter base. The altitude is two less than the shorter base.

  a If *x* represents the shorter base, express in terms of *x* the longer base, the altitude and the area.   [1, 1, 2]

  b If the area of the trapezoid is 32, find the shorter base, the longer base and the altitude.   [4, 1, 1]

33 On each side of a regular hexagon a square is constructed, and the outside vertices of the figure thus formed are joined to form a regular polygon of 12 sides as shown in the figure. If a side of the hexagon is 2, find the area of this 12-sided polygon. [Answer may be left in radical form.]   [10]
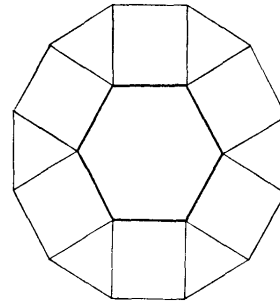
Fill in the following lines:

Name of pupil..Name of school...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 In triangle RST, angle $R = 50°$ and angle $S = 70°$. Name the longest side of the triangle.

1................

2 The complement of angle A contains $50°$. Find the number of degrees in the supplement of angle A.

2................

3 Find the altitude of an equilateral triangle whose side is 8. [Answer may be left in radical form.]

3................

4 A tangent and a secant are drawn to a circle from an external point. The length of the tangent is 6 and the exterior segment of the secant is 4. Find the length of the secant.

4................

5 Find the number of degrees in an interior angle of a regular polygon of nine sides.

5................

6 The side of a regular polygon of n sides is s, and its apothem is a. Express in terms of n, s and a the area of the polygon.

6................

7 Two consecutive angles of a parallelogram are in the ratio $1:5$. Find the number of degrees in the smaller angle.

7................

8 From point K on a circle, chords KL and KM are drawn. If angle LKM is equal to $35°$, find the number of degrees in arc LM.

8................

9 Two perpendicular chords intersect within a circle. Find the sum of one pair of opposite intercepted arcs.

9................

10 One side of a square is 3. Find a diagonal of the square. [Answer may be left in radical form.]

10................

11 Find the length of an arc intercepted by a central angle of $36°$ in a circle whose radius is 20. [Answer may be left in terms of π.]

11................

12 The hypotenuse and a leg of a right triangle are 9 and 6, respectively. Find the shorter segment of the hypotenuse made by the altitude on the hypotenuse.

12................

13 The corresponding altitudes of two similar triangles are 5 and 7. Find the ratio of the perimeters of the two triangles.

13................

14 Corresponding sides of two similar triangles are in the ratio $a:b$. Find the ratio of their areas.

14................

15 In parallelogram $ABCD$, diagonal $AC = 24$ and the perpendicular from B to $AC = 8$. Find the area of the parallelogram.

15................

[3]

[OVER]

16 In triangle ABC, angle C is a right angle. If angle $A = 58°$ and side $AC = 10$, find BC to the *nearest integer*.   16................

17 How many points are there which are equidistant from two parallel lines 4 inches apart and 3 inches from a point on one of the given lines?   17................

Directions (18–22): If the blank in each of the following statements is replaced by one of the words *always, sometimes* or *never,* the resulting statement will be true. Select the word that correctly completes *each* statement and write this word on the line at the right.

18 If two straight lines are cut by a transversal so that one pair of interior angles on the same side of the transversal are supplementary, the lines are ... parallel.   18................

19 Diagonal AC of parallelogram $ABCD$ ... bisects angle A and angle C.   19................

20 As the diameter of a circle increases, the ratio of the circumference to the diameter of the circle ... increases.   20................

21 As the number of sides of a regular polygon inscribed in a circle increases, the apothem of the polygon ... increases.   21................

22 Two right triangles are ... congruent if any two sides of one triangle are equal to the corresponding sides of the other.   22................

23 If the three angles of a triangle are acute, the sum of the two smallest angles is   (a)less than the third angle   (b)equal to the third angle (c)greater than the third angle.   Which is correct (a), (b) or (c)?   23................

Directions (24–25): Leave all construction lines on the paper.

24 Inscribe a square in the given circle whose center is O.
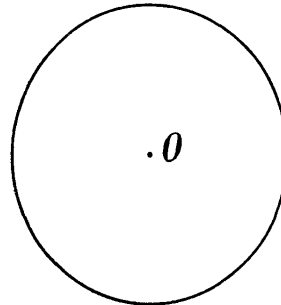
25 Divide line segment AB into two segments in the ratio $m:n$.

$$\underline{\hspace{2cm} m \hspace{2cm}}$$

$$\underline{\hspace{1.5cm} n \hspace{1.5cm}}$$

A ——————————— B

[4]