The University of the State of New York

313TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, August 21, 1951 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1951 or number and length in minutes of lessons taken in the summer of 1951 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1951 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer three questions from part II.

26 Prove that if two sides of a quadrilateral are equal and parallel, the figure is a parallelogram. [10]

27 Prove that if from a point outside a circle a tangent and a secant are drawn to the circle, the tangent is the mean proportional between the secant and its external segment. [10]

28 Pentagon $ABCDE$ is inscribed in a circle and diagonals AC and AD are drawn.

a If chord AB = chord AE and chord AC = chord AD, prove that arc BC = arc ED. [4]

b If a tangent is drawn to the circle at A, prove that this tangent is parallel to CD. [6]

29 In triangle ABC, AB is greater than BC. D is any point in AC and BD is drawn. Prove

a angle BDA is greater than angle ACB [3]

b AB is greater than BD [7]

Part III

Answer one question from part III.

30 In triangle ABC, $AB = 15$, angle $C = 76°$ and angle $A = 42°$. Altitude BD is drawn.

a Find, to the *nearest tenth*, AD, BD and DC. [3, 2, 3]

b Using the results found in *a*, find, to the *nearest integer*, the area of triangle ABC. [2]

31 A circle is inscribed in a regular hexagon whose side is s.

a Express the area of the hexagon in terms of s. [2]

b If the area of the hexagon is $24\sqrt{3}$, find s. [3]

c Find the apothem of the hexagon. [Answer may be left in radical form.] [2]

d Find, to the *nearest integer*, the difference between the area of the hexagon and that of the inscribed circle. [Use $\sqrt{3} = 1.73$ and $\pi = 3.14$.] [3]

Part IV

Answer one question from part IV.

32 If the blank space in each of the following statements is filled by one of the words *always, sometimes* or *never,* the resulting statement will be true. Write on your answer paper the letters *a, b, c, d, e* and opposite *each* write the word that will correctly complete the corresponding statement.

a The diagonals of a rectangle are ... unequal. [2]

b If the diagonals of a quadrilateral bisect each other at right angles, the quadrilateral is ... a square. [2]

c If two sides and an angle of a triangle are given, the triangle is ... determined. [2]

d If two altitudes of a triangle are equal, the triangle is ... isosceles. [2]

e A triangle is inscribed in a circle. If the line segment joining the mid-points of two sides of the triangle is equal to the radius of the circle, the triangle is ... a right triangle. [2]

33 A regular octagon is to be formed by cutting an isosceles right triangle from each corner of a square. Representing a leg of one of the right triangles by x,

a express a side of the octagon in terms of x [2]

b express a side of the square in terms of x [2]

c if a side of the square is 17 inches, find x [Use $\sqrt{2} = 1.4$.] [3]

d find the area of the octagon [3]

[2]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The vertex angle of an isosceles triangle is 140°. Find the number of degrees in an exterior angle formed by extending the base. 1.....

2 An exterior angle of a regular polygon is 45°. Find the number of sides of the polygon. 2.....

3 Angle BAC is inscribed in circle O and radii OB and OC are drawn. If angle BAC is 40°, find the number of degrees in angle BOC. 3.....

4 Two chords intersecting within a circle intercept opposite arcs of 80° and 50°. Find the number of degrees in one of the acute angles formed by the chords. 4.....

5 Two secants drawn to a circle from an external point intercept arcs of 100° and 40°. Find the number of degrees in the angle formed by the secants. 5.....

6 A line parallel to side RS of triangle RST intersects TR at M and TS at N. If the ratio of TM to MR is 2 : 3 and if $TN = 6$, find NS. 6.....

7 CD is the altitude on the hypotenuse of right triangle ABC. If $AD = 8$ and $DB = 18$, find CD. 7.....

8 A chord 14 inches long is 4 inches from the center of the circle. Find the length of the radius. [Answer may be left in radical form.] 8.....

9 In rectangle $ABCD$ diagonal AC is drawn. If BC is half AC, find the number of degrees in angle CAB. 9.....

10 Chords AB and RS intersect at P. If $AP = 15$, $PB = 4$ and $RP = 12$, find PS. 10.....

11 A diagonal of a rectangle makes an angle of 38° with the base. If the diagonal is 20, find the base to the *nearest tenth*. 11.....

12 Find the area of an equilateral triangle whose side is 3. [Answer may be left in radical form.] 12.....

13 Find the area of a rhombus whose diagonals are 7 and 12. 13.....

14 Find the area of a trapezoid whose bases are 5 and 8, and whose altitude is 6. 14.....

[OVER]

15 A side of a square is 3 and a side of another square is 2. Find the ratio of the areas of the squares. 15.....

16 The perimeter of a regular polygon is m and its apothem is d. Express the area of the polygon in terms of m and d. 16.....

17 A right triangle is inscribed in a circle. If the hypotenuse is 20, find the radius of the circle. 17.....

18 Find the radius of a circle whose circumference is 16π. 18.....

19 Find the area of a sector whose central angle is 90° if the radius of the circle is 6. [Answer may be left in terms of π.] 19.....

20 Is statement B the converse of statement A? [Answer *yes* or *no*.]
A A circle can be circumscribed about a regular polygon.
B If a circle can be circumscribed about a polygon, the polygon is regular. 20.....

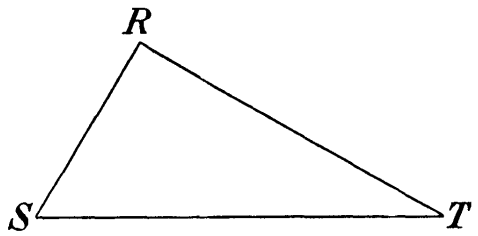
Directions (questions 21–23) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

21 If in triangle CDE, angle $C = 45°$ and CD is the shortest side, then angle D is (*a*) acute (*b*) right (*c*) obtuse 21.....

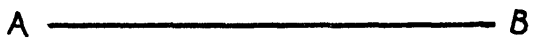
22 A median of a triangle divides it into two triangles which are always (*a*) similar (*b*) equal (*c*) congruent 22.....

23 The locus of points equidistant from two given points is a (*a*) point (*b*) line (*c*) circle 23.....

24 Find the center of the circle that can be inscribed in triangle SRT.



25 If line segment AB represents the perimeter of an equilateral triangle, find the line segment that represents a side of the triangle.



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