The University of the State of New York

304TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, August 24, 1948 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1948 or number and length in minutes of lessons taken in the summer of 1948 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1948 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer three questions from part II.

26 Prove that if two sides of a quadrilateral are equal and parallel, the figure is a parallelogram. [10]

27 Prove that an angle formed by a tangent and a secant intersecting outside a circle is measured by one-half the difference of the intercepted arcs. [10]

28 Triangle ABC is similar to triangle $A'B'C'$. The bisectors of angles B and B' intersect AC and $A'C'$ in D and D' respectively. Prove: $BD : BC = B'D' : B'C'$ [10]

29 Prove that the altitudes to the legs of an isosceles triangle are equal. [10]

Part III

Answer one question from part III.

30 The radius of circle O is 12 inches and the length of minor arc AB is 4π inches. Chord AB and radii OA and OB are drawn.

a Find the number of degrees in angle AOB. [3]

b Find the area of sector AOB. [Answer may be left in terms of π.] [2]

c Find the area of triangle AOB. [Answer may be left in radical form.] [2]

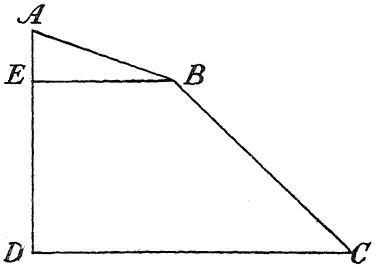
d Find to the *nearest square inch* the area of segment AB. [$\pi = 3.14$ and $\sqrt{3} = 1.73$] [3]

31 An owner had to calculate the floor space in his garage in order to get a renting permit. The floor plan of the garage is given by the diagram at the right.

In this drawing, $AB = 60$ ft, $AD = 100$ ft, $DC = 160$ ft, angle $A = 77°$ and angle $D = 90°$. BE is drawn perpendicular to AD.

a Find AE and BE to the *nearest tenth of a foot.* [6]

b Using the results found in answer to *a*, find the area of the floor space to the *nearest square foot.* [4]



Part IV

Answer one question from part IV.

32 Each of the five parts of this question contains a statement that can be correctly completed by two and only two of the given choices. Write the numbers 1 to 5 on your answer paper and after *each* indicate the correct answer to the corresponding question by writing *only two* of the letters *a, b, c, d.*

(1) A triangle must be a right triangle if (*a*) its sides are in the ratio 3:4:5 (*b*) its angles are in the ratio 3:4:5 (*c*) two of its angles are complementary (*d*) an altitude divides it into two congruent triangles [2]

(2) There is always a point that is equidistant from the sides of a (*a*) triangle (*b*) parallelogram (*c*) rhombus (*d*) rectangle [2]

(3) Triangle ABC is inscribed in a circle whose center is O. If triangle ABC is (*a*) acute, then O is outside ABC (*b*) right, then O is on a side of ABC (*c*) equilateral, then central angles AOB, BOC and AOC are acute (*d*) isosceles, then the bisector of the vertex angle passes through O [2]

(4) A polygon must be regular if (*a*) it is inscribed in a circle and is equilateral (*b*) each of its angles contains 60° (*c*) it is inscribed in a circle and is equiangular (*d*) a circle can be inscribed in it and a circle can be circumscribed about it [2]

(5) Only one triangle ABC can be constructed if the given parts are (*a*) the three angles (*b*) the three sides (*c*) side AB, angle A and angle C (*d*) side AB, side BC and angle A [2]

33 From any point P in base BC of isosceles triangle ABC, PR and PS are drawn perpendicular to legs AB and AC respectively. Altitude BH and line AP are drawn.

a In terms of lines given above, write an expression for the area of each of the triangles ABP, APC and ABC. [3]

b Write an equation showing how these areas are related. [2]

c Using this equation, show that $PR + PS = BH$ [5]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Find the side of a square whose diagonal is 12. [Answer may be left in radical form.] 1.....

2 Find the altitude of an equilateral triangle whose side is 6. [Answer may be left in radical form.] 2.....

3 Chord AC is 8 inches long. If arc AC contains 60°, how many inches are there in the radius of the circle? 3.....

4 How many degrees are there in each angle of a regular pentagon? 4.....

5 Triangle ABC is a right triangle with the right angle at C. From vertex A a line is drawn to any point D in the opposite side BC. Name the longest side of triangle ABD. 5.....

6 The apothem of a regular polygon is 10 and its perimeter is p. Express the area of the polygon in terms of p. 6.....

7 Find the area of a rhombus whose diagonals are 6 and 10. 7.....

8 Find the area of a trapezoid whose bases are 8 and 20 and whose altitude is 7. 8.....

9 Find the area of a right triangle whose legs are 6 and 7. 9.....

10 The circumference of a circle is 16π. Find its area. [Answer may be left in terms of π.] 10.....

11 At a distance of 200 feet from the foot of a flagpole, the angle of elevation of the top of the pole was observed to be 22°. Find to the *nearest foot* the height of the pole. 11.....

12 In right triangle ABC, altitude CD is drawn to hypotenuse AB. If $AD = 3$ and $AB = 12$, find AC. 12.....

13 In circle O, chord AB intersects chord CD at M. If $AM = 16$, $MB = 3$ and $CM = 8$, find MD. 13.....

14 From a point A outside a circle, tangent AB and secant ACD are drawn. If $AB = 10$ and $AD = 20$, find the length of the external segment AC. 14.....

15 Corresponding sides of two similar triangles are in the ratio 1:4. Find the ratio of their areas. 15.....

Directions (questions 16–19) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b* or *c*.

16 Two figures must be similar if they are (*a*) rectangles (*b*) equilateral triangles (*c*) regular polygons 16.....

17 If a median of a triangle is equal to one-half the side to which it is drawn, the given triangle must be (*a*) right (*b*) isosceles (*c*) equilateral 17.....

18 If one of the exterior angles of a triangle is acute, the triangle must be (*a*) acute (*b*) right (*c*) obtuse 18.....

19 By definition the number π (*a*) is a constant and is equal to the ratio of the circumference of a circle to its diameter (*b*) is a constant and is equal to the ratio of the circumference of a circle to its radius (*c*) varies with the size of the circle 19.....

Directions (questions 20–23) — If the blank in each statement is replaced by one of the words *always, sometimes* or *never,* the resulting statement is true. Select the word that will correctly complete *each* statement and write the word on the line at the right.

20 If a statement is true, a converse of the statement is ... true. 20.....

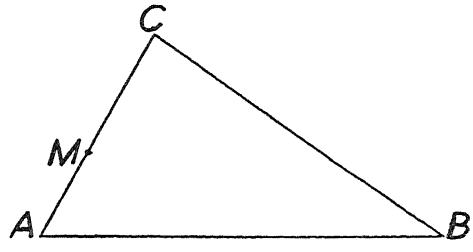
21 As the number of sides of a polygon increases, the sum of its exterior angles ... increases. 21.....

22 If an acute angle changes so that its sine increases, its tangent ... increases. 22.....

23 If the diagonals of a quadrilateral bisect each other, the quadrilateral is ... a rhombus. 23.....

Directions (question 24–25) — Leave all construction lines on your paper.

24 On CB, find a point which will divide CB into segments having the ratio $CM : MA$



25 Construct the locus of points which are the same distance from the end points of line segment AB.