The University of the State of New York

301ST HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Wednesday, August 20, 1947 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1947 or number and length in minutes of lessons taken in the summer of 1947 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1947 or an equivalent program of tutoring approved in advance by the Department is required.

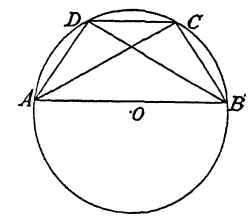
Part II

Answer three questions from part II.

26 Prove that the opposite sides of a parallelogram are equal. [10]

27 Quadrilateral $ABCD$ is inscribed in a circle and diagonal AC = diagonal BD. Prove:

a Arc AD = arc BC [6]
b Side AD = side BC [1]
c $\angle BDC = \angle ABD$ [2]
d Side $DC \parallel$ side AB [1]



28 Prove that the area of a triangle is equal to one half the product of its base and its altitude. [10]

29 Triangle ABC is inscribed in a circle. BH is an altitude of the triangle and BD is a diameter of the circle. Prove: $AB : BH = BD : BC$ [10]

Part III

Answer one question from part III.

30 In rhombus $ABCD$, angle $A = 72°$, diagonal $AC = 20$ and diagonals AC and BD intersect at E.

a Find BE correct to the *nearest tenth.* [7]

b Using the result found in answer to *a,* find the area of $ABCD$. [3]

31 In circle O, radius $OA = 12$, arc $AB = 60°$ and lines AB and OB are drawn. Find the area of

a Sector AOB correct to the *nearest tenth* [Use $\pi = 3.14$] [4]

b Triangle AOB correct to the *nearest tenth* [$\sqrt{3} = 1.73$] [4]

c Segment AB of the circle [2]

Part IV

Answer one question from part IV.

32 Each of the five parts of this question is a statement that may be correctly completed by *one or more* of the given choices. Write the numbers (1) to (5) on your answer paper and after *each* indicate the correct answer or answers to the corresponding question by writing one or more of the letters *a, b, c, d.* [10]

[In each of the five parts of the question, one credit will be allowed for each correct choice made and one credit will be deducted for each incorrect choice. The minimum credit on each part will be 0.]

(1) A regular polygon is inscribed in a given circle. As the number of sides of this polygon increases (*a*) the sum of the exterior angles does not change, (*b*) the radius of the polygon increases, (*c*) the perimeter of the polygon increases, (*d*) the apothem of the polygon increases

(2) If P is any point inside a circle whose center is O, (*a*) two equal chords can be drawn through P, (*b*) line OP bisects at least two chords through P, (*c*) the shortest chord through P is perpendicular to the diameter which passes through P, (*d*) the length of the longest chord through P changes as the position of P changes

(3) In the regular hexagon $ABCDEF$, (*a*) the apothem is one half the radius of the polygon, (*b*) diagonal BE is twice side BC, (*c*) quadrilateral $BCEF$ is a square, (*d*) all the diagonals of the hexagon are equal

(4) Base AB of triangle ABC is fixed in length and position. The locus of vertex C is a circle (*a*) if angle C is a right angle, (*b*) if C is equidistant from A and B, (*c*) if C is at a given distance from AB, (*d*) if the median from C is equal to a given line segment

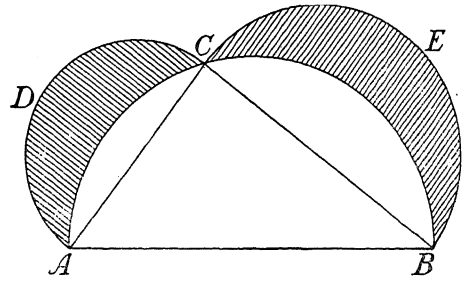
(5) Quadrilateral $ABCD$ is inscribed in a circle. The quadrilateral (*a*) will be a rectangle if opposite sides are parallel, (*b*) may be a rhombus, (*c*) is an isosceles trapezoid if only two sides are parallel, (*d*) always has one pair of equal angles

33 In the diagram below, angle ACB is a right angle. Semicircles ADC and CEB are drawn with AC and CB as diameters, respectively. Semicircle ACB is drawn with AB as a diameter. If AC is represented by $6x$ and CB by $8x$,

a Find, in terms of x and π, the area of semicircles ADC and CEB [2]

b Find, in terms of x and π, the area of semicircle ACB [3]

c Show that the area of the shaded portion equals the area of the triangle [5]



PLANE GEOMETRY

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Two sides of a parallelogram are 10 and 12 and they include an angle of 30° ; find the altitude on side 12. 1.....

2 From a point outside a circle a tangent and a secant are drawn. The tangent is 15 and the entire secant is 25; find the external segment of the secant. 2.....

3 A chord whose length is 12 inches is 8 inches from the center of the circle. How many inches are there in the radius? 3.....

4 Find the area of the trapezoid whose bases are 5 and 9 and whose altitude is 6. 4.....

5 The apothem of a regular polygon is 5 and its perimeter is 40; find its area. 5.....

6 The altitude on the hypotenuse of a right triangle divides the hypotenuse into segments 2 and 8; find the altitude. 6.....

7 Chords AB and CD in a circle intersect at M. If $AM = 6$, $MB = 8$ and $CM = 12$, find MD. 7.....

8 The area of a circle is 36π. Find a side of the square circumscribed about the circle. 8.....

9 From an external point P, a tangent PA and a secant PBC are drawn to a circle. The secant passes through the center of the circle. Arc $CA = 110°$. Find the number of degrees in angle P. 9.....

10 A diagonal of a square is $3\sqrt{2}$; find a side of the square. 10.....

11 How many degrees are there in *each* angle of a regular polygon of 8 sides? 11.....

12 Regular pentagon $ABCDE$ is inscribed in circle O. Diagonals AD and CE intersect at P; find the number of degrees in angle CPD. 12.....

Directions (questions 13–14) — If the blank in each statement is replaced by one of the words *always, sometimes* or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

13 If two angles are supplementary they will ... be adjacent. 13.....

14 If two isosceles triangles have a base angle of one equal to a base angle of the other, the triangles will ... be similar. 14.....

Directions (questions 15–21) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b* or *c.*

15 As an angle increases, its complement (*a*) increases, (*b*) decreases, (*c*) remains the same 15.....

16 Corresponding sides of two similar polygons are in the ratio 1:4. Their areas are in the ratio (*a*) 1:16, (*b*) 1:4, (*c*) 1:2 16.....

17 The locus of points equidistant from three points on a circle is (*a*) a point, (*b*) a line, (*c*) two lines 17.....

[OVER]

18 From external point A, tangents AB and AC are drawn to circle O. Radii OB and OC are drawn. Then angle A and angle O must be (a) supplementary, (b) equal, (c) right 18.....

19 The center of the circle circumscribing an obtuse triangle is (a) inside the triangle, (b) on one side of the triangle, (c) outside the triangle 19.....

20 Triangle ABC has a right angle at C. If $\sin A = \frac{1}{3}$, then (a) $\frac{BC}{BA} = \frac{1}{3}$,

(b) $\frac{AC}{AB} = \frac{1}{3}$, (c) $\frac{CB}{CA} = \frac{1}{3}$ 20.....

21 Which of the following is a *good* definition?

a A trapezoid is a quadrilateral with two sides parallel.

b A quadrilateral is a polygon which has four sides and four angles.

c A median of a triangle is the line segment drawn from a vertex to the mid-point of the opposite side. 21.....

22 In triangle ABC, $AB = AC$ and $\angle B = 55°$. Which is the longer, side BC or side BA? 22.....

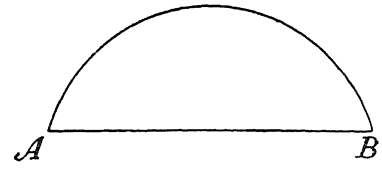
23 Is statement *b* the converse of statement *a*? [Answer *yes* or *no*.]

a A resident of New York State is a resident of the U. S. A.

b A resident of the U. S. A. is a resident of New York State. 23.....

Directions (questions 24–25) — Leave all construction lines on your paper.

24 Find by construction the mid-point of arc AB.



25 Divide line segment AB into segments having the ratio $m : n$.