The University of the State of New York

279TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Thursday, August 22, 1940 — 8.30 to 11.30 a. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II, III and IV

Write at top of first page of answer paper to groups II, III and IV (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1940, (*d*) author of textbook used.

The minimum time requirement is five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1940 is required.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–10) — Write on the dotted line at the right of each question the expression which, when inserted in the corresponding blank, will make the statement true.

1 In the right triangle ABC, if angle B equals 30° and AC equals 2 inches, then the hypotenuse AB equals ... inches. 1.....

2 If the mid-points of two adjacent sides of a rhombus are joined, the triangle formed is .... 2.....

3 If the area of a circle is 49π, the circumference in terms of π is .... 3.....

4 If the altitude upon the hypotenuse of a right triangle divides the hypotenuse into segments of 18 and 32, then the shorter leg of the given triangle is .... 4.....

5 If in right triangle ABC angle $C = 90°$, angle $A = 70°$ and $AB = 50$, then AC correct to the *nearest integer* is .... 5.....

6 If the diagonals of a rhombus are 6 and 8, then the area of the rhombus is .... 6.....

7 Point P is a distance of 6 from the center of a circle whose radius is 10; the product of the segments of any chord drawn through P is .... 7.....

8 The radius of a circle is 9 and the angle of a sector of this circle is 40°; the area of this sector in terms of π is .... 8.....

9 Two triangles are similar and the area of the first triangle is four times the area of the second. If a side of the first triangle is 8, then the corresponding side of the second is .... 9.....

10 The length of a tangent drawn from a point 3 inches from a circle whose radius is 12 inches is ... inches. 10.....

Directions (questions 11–16) — Indicate the correct answer to each question by writing on the dotted line at the right the letter *a*, *b* or *c*.

11 Vertical angles are always (*a*) acute, (*b*) equal or (*c*) supplementary. 11.....

12 An exterior angle at the base of an isosceles triangle is always (*a*) an acute angle, (*b*) an obtuse angle or (*c*) a right angle. 12.....

13 All quadrilaterals whose equal diagonals bisect each other are (*a*) rectangles, (*b*) squares or (*c*) rhombuses. 13.....

14 The angle that is measured by one half the difference of its intercepted arcs has its vertex (*a*) within the circle, (*b*) on the circle or (*c*) outside the circle. 14.....

15 The two chords that form the sides of an angle inscribed in a semicircle are always (*a*) equal, (*b*) unequal or (*c*) perpendicular to each other. 15.....

16 The number of circles that can be tangent to two intersecting lines is (*a*) two, (*b*) four or (*c*) unlimited. 16.....

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Directions (questions 17–21) — Indicate whether each of the following statements is *always true, sometimes true* or *never true* by writing the word *always, sometimes* or *never* on the dotted line at the right.

17 If one angle of a triangle is 60° and the other two angles are unequal, the side opposite the 60° angle is the longest side of the triangle. 17.....

18 If a polygon is equilateral, it is equiangular. 18.....

19 If in the same circle or in equal circles two chords are equal, they are equidistant from the center. 19.....

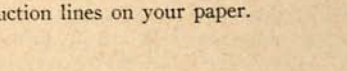
20 Similar triangles are congruent triangles. 20.....

21 If the number of sides in a polygon is increased by 2, the sum of the exterior angles of this polygon, made by producing each of its sides in succession, remains the same. 21.....

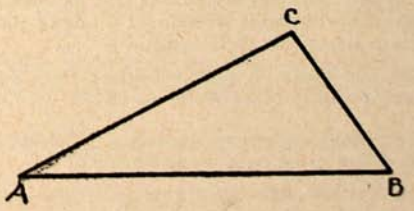
22 Is the converse of the following theorem true: "Two parallel lines intercept equal arcs on a circle"? [Answer *Yes* or *No*.] 22.....

Directions (questions 23–25) — Leave all construction lines on your paper.

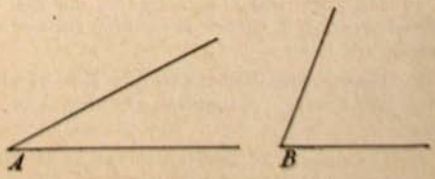
23 Find the locus of the centers of all circles which will pass through the two points A and B.



24 Find the center of the circle that may be inscribed in the triangle ABC.



25 Angles A and B are two angles of triangle ABC. Construct angle C.



See instructions for groups II, III and IV on page 1.

Group II

Answer two questions from this group.

26 Prove that if two sides of a quadrilateral are equal and parallel, the figure is a parallelogram. [10]

27 Prove that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [10]

28 *A*, *B*, *C* and *D* are four points taken consecutively on a circle and so located that arc *BC* is twice each of the arcs *AB* and *CD*. Chords *AC* and *BD* are drawn intersecting in *M* and chord *DC* is drawn. Prove that triangle *DCM* is isosceles. [10]

Group III

Answer two questions from this group.

29 A trapezoid is inscribed in a circle whose radius is 17 inches, the center of the circle lying within the trapezoid. Find the area of the trapezoid if its bases are 8 inches and 15 inches from the center. [10]

30 The diameter of a circle is 20 inches.

a Find the length of the apothem and the area of a regular inscribed hexagon. [5]

b Find the length of a side and the area of an inscribed equilateral triangle. [5] [Both answers may be left in radical form.]

31 The altitude of a triangle is 12 inches and it divides the vertex angle into two angles of 31° and 45°.

a Find the lengths of the segments of the base. [6]

b Find, correct to the *nearest square inch*, the area of the triangle. [4]

Group IV

Answer one question from this group.

32 Given parallelogram *ABCD* with diagonal *BD*. A line from *C* cuts *BD* in *E* and *AB* in *F*.

a Prove triangle *BEF* similar to triangle *CDE*. [3]

b If *F* is the mid-point of *AB*, what is the relation between *BE* and *ED*? Give reason. [3]

c Using your answer to *b*, find the area of triangle *CED* if the area of triangle *BEC* is 100 square inches. [4]

33 The medians of a triangle are 18, 15 and 15. Find the area of the triangle formed by joining the feet of the three medians. [10]

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