The University of the State of New York

264TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, August 20, 1935 — 8.30 to 11.30 a. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1935, (*d*) author of textbook used.

The minimum time requirement previous to entering summer high school is five recitations a week for a school year.

For those pupils who have met the time requirement previous to entering summer high school the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1935 is required.

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See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

21 Prove that an angle formed by two chords intersecting within a circle is measured by one half the sum of the intercepted arcs. [10]

22 Prove that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [10]

23 Diagonal AC of quadrilateral $ABCD$ divides the figure into two triangles equal in area but not congruent. Prove that points D and B are equidistant from AC. [10]

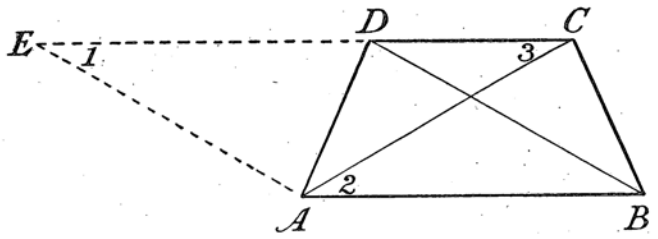
24 Diameter CB of circle O is extended through B to P and a tangent from P meets the circle at A. AC and AB are drawn and AC equals AP. Prove that AB equals PB. [10]

25 If the diagonals of a quadrilateral are equal and two sides are parallel, then the other two sides are equal.

Given quadrilateral $ABCD$ with sides AB and DC parallel and diagonals AC and BD equal.

To prove: $AD = BC$

Construction: Draw AE parallel to BD and meeting CD produced at E.



Below are given the statements for a proof of the above theorem but they are not arranged in logical order. *Do not copy the above material.* Rearrange the statements in logical order on your answer paper and assign a reason for *each* statement. [10]

$\therefore ED = AB$; $EA = DB$

$\therefore \angle 1 = \angle 2$

$AC = BD$

$ABDE$ is a parallelogram.

$\angle 1 = \angle 3$

$\angle 2 = \angle 3$

$\therefore AD = BC$

$\triangle ADE \cong \triangle ABC$

$\therefore EA = AC$

Group III

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Irrational results may be left in the form of π and radicals unless otherwise stated.

26 In a certain circle the mid-point of a chord 16 inches long is 4 inches from the minor arc subtended by the chord.

a Find the perimeter of the triangle formed by the radii drawn to the extremities of the chord. [7]

b Find the area of the triangle. [3]

27 The diagonals of a rhombus are 18 and 24. Find the radius of the circle inscribed in the rhombus. [10]

28 An equilateral triangle and a hexagon are circumscribed about a circle whose radius is $10\sqrt{3}$. Find the difference in area between the triangle and the hexagon. [Answer may be left in radical form.] [10]

29 A and B are two points at opposite ends of a pond whose length AB some boy scouts wish to determine. To do this they locate a point C 100 feet from each of the two points A and B and measure the angle BAC, which they find to be 58°. What is the length of AB correct to the *nearest foot*? [10]

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (question 1–17) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true. No work need be shown.

1 The difference between the supplement and the complement of any acute angle is an angle of ... degrees. *Ans*.....

2 If one acute angle of a right triangle is 30°, the ratio of the shorter leg to the hypotenuse is .... *Ans*.....

3 If in the same circle or in equal circles two chords are unequal, the ... chord is nearer the center. *Ans*.....

4 In a certain quadrilateral $ABCD$, angle A and angle C are right angles; then angle B + angle D = ... degrees. *Ans*.....

5 If the sum of the interior angles of a certain polygon equals the sum of the exterior angles of the same polygon, the number of sides of the polygon must be .... *Ans*.....

6 Finding a point in a triangle equidistant from the three sides involves bisecting two ... *Ans*.....

7 The circumference of a circle is 18π; its area in terms of π is .... *Ans*.....

8 A line 4 inches long joins the mid-points of two sides of a triangle; the length of the third side is ... inches. *Ans*.....

9 PA and PB are tangents to circle O from point P. If angle $APB = 60°$ and $PA = 8$ inches, the length of chord AB is ... inches. *Ans*.....

10 The bases of a trapezoid are 8 feet and 10 feet and its area is 45 square feet; the altitude of the trapezoid is ... feet. *Ans*.....

11 If the area of a sector is $\frac{2}{9}$ of the area of the whole circle, the angle of the sector contains ... degrees. *Ans*.....

12 If side AB of triangle ABC is extended through B and forms an exterior angle of 75°, then side ... is the longest side of the triangle. *Ans*.....

13 Two triangles are similar and the area of the first triangle is four times the area of the second. If a side of the first triangle is 8, the corresponding side of the second triangle is .... *Ans*.....

14 The length of a tangent from a point 4 inches from a circle whose diameter is 12 inches is ... inches. *Ans*.....

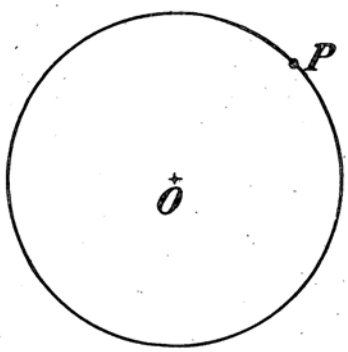
15 In a triangle a series of straight lines parallel to the base terminate in the other two sides of the triangle. The locus of the mid-points of these lines is the ... of the triangle. *Ans*.....

16 If the altitude upon the hypotenuse of a right triangle divides the hypotenuse into segments of 4 inches and 9 inches, then the altitude is ... inches. *Ans*.....

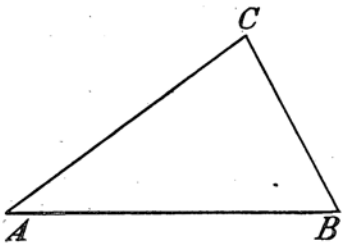
17 In parallelogram $ABCD$, if E is the mid-point of side BC and lines AE and DE are drawn, then triangle ADE is exactly ... of parallelogram $ABCD$. *Ans*.....

Directions (questions 18–20) — Leave all construction lines on the paper.

18 Construct a tangent to the circle O at the point P.



19 On DE as a base corresponding to AB, construct a triangle similar to the given triangle ABC.



20 Construct the locus of points equidistant from the parallel lines a and b.

a

b