

University of the State of New York
High School Department
 170TH EXAMINATION
PLANE GEOMETRY
 August 1901 — Three hours, only

Answer eight questions but no more, including at least one from each of the three divisions. If more than eight are answered only the first eight answers will be considered. Draw carefully and neatly each figure in construction or proof, using letters instead of numerals. Arrange work logically. Each complete answer will receive 12½ credits. Papers entitled to 75 or more credits will be accepted.

First 1 Prove that the perpendicular is the shortest line that **division** can be drawn to a straight line from an external point.

2 Complete and demonstrate the following: an angle formed by two secants, two tangents, or a tangent and a secant, drawn to a circle from an external point, is measured by . . .

3 Prove that two triangles whose sides are respectively parallel or respectively perpendicular are similar.

4 Complete and demonstrate the following: in any triangle the square of the side opposite an acute angle is equal to . . .

5 Prove that a circle can be circumscribed about any regular polygon.

Second 6 Find the ratio of an angle of a regular octagon to **division** the exterior angle formed by producing one of the sides of the angle.

7 The ratio of the radii of two circles, one of which lies wholly within the other, is 3 to 4; what part of the larger circle is included between the two circumferences?

8 The bases of a trapezoid are 30 in. and 12 in. respectively, the altitude is 9 in.; find the altitudes of the two triangles formed by producing the legs of the trapezoid till they meet.

9 Two tangents, each 6 in. long, drawn from an exterior point to a circle, intercept an arc of 60° ; find the distance of the exterior point from the center of the circle.

10 On a given line 5 in. long a segment of a circle is described which contains an angle of 45° ; find the length of the radius of the circle.

Third 11 Prove that any line drawn through the middle **division** point of a diagonal of a parallelogram divides the parallelogram into two equal parts.

12 Show how to construct a square equivalent to a given rhomboid.

13 Prove that the bisector of the vertical angle of a triangle forms with the bisector of one of the exterior angles at the base, an angle equal to $\frac{1}{2}$ the other angle at the base.

14 On the sides of the triangle ABC equilateral triangles ABD , BCF and ACE are drawn; prove that lines FA , DC and BE are equal.

15 Show how to inscribe a square in a given semicircle.