

## High School Department

165TH EXAMINATION

## PLANE GEOMETRY

August 1900 — Three hours, only

*Answer eight questions but no more, including at least one from each of the three divisions. If more than eight are answered only the first eight answers will be considered. Draw carefully and neatly each figure in construction or proof, using letters instead of numerals. Arrange work logically. Each complete answer will receive 12½ credits. Papers entitled to 75 or more credits will be accepted.*

**First division** 1 Define five of the following: *variable, heptagon, convex polygon, perimeter, re-entrant angle, demonstration, secant.*

2 Prove that two right triangles are equal if a side and the hypotenuse of one are equal respectively to a side and the hypotenuse of the other.

3 Complete and demonstrate the following: an angle formed by two secants or two tangents intersecting without the circumference is measured by . . .

4 Prove that two triangles which have their sides respectively parallel or respectively perpendicular are similar.

5 Prove that two regular polygons of the same number of sides are similar.

**Second division** 6 Tangents 15 inches long are drawn from a point to a circle whose diameter is 16 inches; find the length of the chord joining the points of contact.

7 The radius of a circle is  $r$ ; find the side of the inscribed octagon.

8 The sides of a triangle are 14 feet and 10 feet and its base is 8 feet; how far must the base be produced to meet the bisector of the exterior angle formed by producing one of the sides through the vertex?

9 One side of an angle of  $30^\circ$  inscribed in a circle is a diameter 12 inches long; find the area included by the sides of the angle and its intercepted arc.

10 The area of a triangle is 30 square inches, its base is 12 inches; find the area, base and altitude of the triangle cut off by a line joining the middle points of the sides of the triangle.

**bird** 11 Prove that any two altitudes of a triangle are in-  
**vision** versely proportional to their corresponding bases.

12 Prove that lines drawn through the vertices of a quadrilateral parallel to the diagonals form a parallelogram twice as large as the quadrilateral.

13 Prove that the line joining the middle points of the diagonals of a trapezoid is equal to half the difference of the bases.

14 Prove that the areas of two triangles which have an angle of the one supplementary to an angle of the other are to each other as the products of the sides including the equal angles.

15 The radius of a circle is  $a$ ; show how to construct a concentric circle whose area will be three times the area of the given circle.