The University of the State of New York

321ST HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Monday, June 21, 1954 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

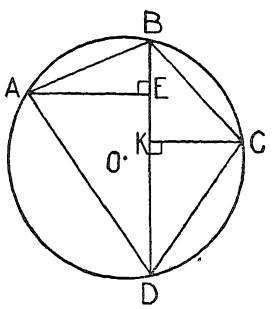
The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove: Tangents drawn to a circle from an external point are equal. [10]

27 In the figure at the right, quadrilateral $ABCD$ is inscribed in circle O. Perpendiculars are drawn to diagonal BD from A and C meeting BD at E and K respectively. If arc AB is equal to arc BC, prove that $ED \times DC = AD \times KD$. [10]



28 Prove: If two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [10]

29 In parallelogram $ABCD$, M is the mid-point of side DC. Line AM extended meets BC extended at K.

a Prove that triangle ADM is congruent to triangle CMK. [4]

b Prove that triangle AKB is equal in area to parallelogram $ABCD$. [6]

30 AB is a chord of a circle and C is the mid-point of minor arc AB. Chord CD is drawn intersecting AB at E. Chords BD and BC are drawn. Prove that if BE is greater than EC, then BD is greater than BC. [10]

Part III

Answer two questions from part III. All work, including computation, should be shown.

31 Triangle ABC is inscribed in a circle. Arcs AB, BC and CA are represented by $x + 70°$, $2x + 20°$ and $3x - 30°$ respectively.

a Find x. [3]

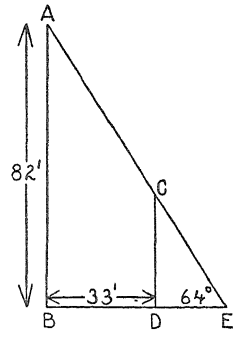
b Show that triangle ABC is equilateral. [3]

c If the bisector of angle B and the tangent to the circle at C when extended meet in S, find the number of degrees in angle S. [4]

32 In the figure at the right, AB is a tower and CD is a flagpole, both of which are standing on level ground. The height of the tower is 82 feet and the distance (BD) from the foot of the pole to the foot of the tower is 33 feet. If the angle of elevation of the sun (angle AEB) is 64°, find to the *nearest foot*

a the length of the shadow (BE) of the tower [5]

b the height of the pole [5]

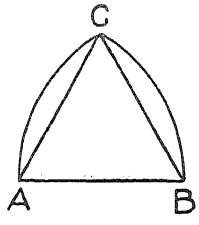


33 In the figure at the right, arc CB is the arc of a circle with A as the center, arc AC is the arc of a circle with B as the center, and $AB = 12$.

a Find the area of sector BAC. [Answer may be left in terms of π.] [3]

b Find the area of the segment bounded by chord BC and arc BC. [Answer may be left in radical form and in terms of π.] [4]

c Find the area of the entire figure. [Answer may be left in radical form and in terms of π.] [3]



34 The difference between the areas of two similar triangles is 18 square feet.

a If A represents the area of the smaller triangle, represent the area of the larger triangle in terms of A. [1]

b If the ratio of the area of the larger triangle to the area of the smaller triangle is 4 : 1, write an equation that can be used to find A. [2]

c Find A. [3]

d If one side of the smaller triangle is 5 feet, find the corresponding side of the larger triangle. [4]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The radius of a circle is 8. Find the circumference of the circle. [Answer may be left in terms of π.] 1.....

2 The altitude of a trapezoid is 9 and the bases are 6 and 18. Find the area of the trapezoid. 2.....

3 The diagonals of a rhombus are 12 and 16. Find a side of the rhombus. 3.....

4 The side of a regular polygon of 8 sides is s and the apothem is a. Express the area of the polygon in terms of a and s. 4.....

5 Two parallel lines are cut by a transversal. One of the two interior angles on the same side of the transversal is 30° more than the other. Find the number of degrees in the smaller angle. 5.....

6 A secant and a tangent are drawn to a circle from an external point. If the tangent is 12 and the external segment of the secant is 6, find the secant. 6.....

7 Chords AB and CD intersect within a circle at E. $AE = EB$, $CE = 4$ and $ED = 9$. Find EB. 7.....

8 Tangents are drawn to a circle from an external point. If the smaller arc intercepted by the tangents contains 110°, find the number of degrees in the angle formed by the tangents. 8.....

9 In right triangle ABC, CD is the altitude upon the hypotenuse. If $AD = 9$ and $DB = 3$, find BC. 9.....

10 On level ground a vertical pole 9 feet tall casts a shadow 3 feet long at the same time that a tree casts a shadow 14 feet long. Find the height of the tree in feet. 10.....

11 AB is the base of parallelogram $ABCD$. If $AD = 10$ and angle $A = 60°$, find the altitude to side AB. [Answer may be left in radical form.] 11.....

12 A square is inscribed in a circle whose diameter is 8. Find the area of the square. 12.....

13 The sum of the interior angles of a polygon is 1800°. How many sides has the polygon? 13.....

14 The base of an isosceles triangle is 20 inches long and a base angle is 57°. Find to the *nearest inch* the length of the altitude to the base. 14.....

15 In rectangle $ABCD$, E is the mid-point of DC and F is the mid-point of AD. If $FE = 7$, find diagonal AC. 15.....

16 Is statement B the converse of statement A? [Answer *yes* or *no*.]

A The three altitudes of an equilateral triangle are equal.

B If the three altitudes of a triangle are equal, the triangle is equilateral. 16.....

[OVER]

Directions (17–19): For *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

17 If two polygons have their corresponding angles equal, the polygons are similar. 17.....

18 If two triangles have equal bases and equal altitudes, the triangles are congruent. 18.....

19 If two opposite angles of a quadrilateral are right angles, a circle can be circumscribed about the quadrilateral. 19.....

Directions (20–23): Indicate the correct completion for *each* of the following by writing the letter *a*, *b* or *c* on the line at the right.

20 In triangle ABC, if $AB = BC$ and angle $A = 50°$, (*a*) AC is less than AB (*b*) AC is equal to AB (*c*) AC is greater than AB 20.....

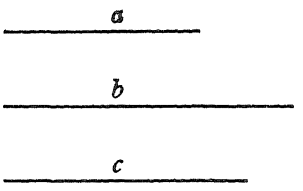
21 C is a point on line l. The number of points 3 inches from C and 2 inches from l is (*a*) 1 (*b*) 2 (*c*) 4 21.....

22 All residents of this state who are registered voters are over 21 years of age. If John is a resident of this state, it is correct to conclude that (*a*) if John is over 21, he is a registered voter (*b*) if John is a registered voter, he is over 21 (*c*) if John is not a registered voter, he is not over 21 22.....

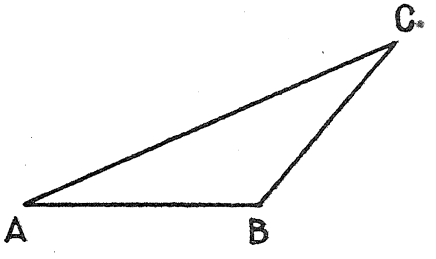
23 Diagonals AC and BD of quadrilateral $ABCD$ intersect at E. $ABCD$ is a parallelogram if (*a*) $AE = BE$ and $DE = EC$ (*b*) $AB = BC$ and $CD = DA$ (*c*) $AE = EC$ and $BE = ED$ 23.....

Directions (24–25): Leave all construction lines on the paper.

24 Construct x such that $a : b = c : x$.



25 Construct the altitude to side AB of triangle ABC.



FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
PLANE GEOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 20–23, allow credit if the pupil has written the correct answer instead of the letter *a, b* or *c*.

(1) 16π
(2) 108
(3) 10
(4) $4as$
(5) 75
(6) 24
(7) 6
(8) 70
(9) 6
(10) 42
(11) $5\sqrt{3}$
(12) 32
(13) 12
(14) 15
(15) 14
(16) yes
(17) false
(18) false
(19) true
(20) *c*
(21) *c*
(22) *b*
(23) *c*