The University of the State of New York

318TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Monday, June 15, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

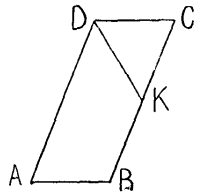
The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove: If two sides of a quadrilateral are equal and parallel, the figure is a parallelogram. [10]

27 In parallelogram $ABCD$, side AB is shorter than side BC. K is a point on BC such that DK bisects angle ADC and BK equals AB.

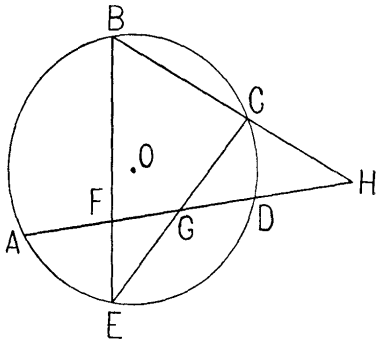


a Prove: $CK = CD$. [6]

b Prove that K is the mid-point of BC. [4]

28 Prove: If two chords intersect within a circle, the product of the segments of one is equal to the product of the segments of the other. [10]

29 In the figure at the right, HCB and HDA are secants to circle O. Chords EB and EC intersect AH in F and G respectively. Arc AB : Arc BC : Arc $CD = 3:2:1$.



a Letting n equal the number of degrees in arc CD, find in terms of n the number of degrees in arcs BC and AB. [1]

b Prove that angle E equals angle H. [4]

c Prove that triangle GEF is similar to triangle CGH. [3]

d If CH is 12, EG is 10 and EF is 8, find GH. [2]

30 In parallelogram $ABCD$, E is a point on diagonal AC. Through E a line is drawn parallel to AB meeting AD in F and BC in K. Through E another line is drawn parallel to AD meeting AB in G and DC in H.

a Prove that triangle HEC is congruent to triangle KEC. [7]

b Prove that the area of quadrilateral $AEHD$ is equal to the area of quadrilateral $AEKB$. [3]

Part III

Answer two questions from this part.

31 Chord AB in circle O is 16 inches long. C is the mid-point of minor arc AB, and diameter CE intersects chord AB in D. CD is 4 inches long.

a Find the length of DE. [6]

b Find the length of a diameter of the circle. [1]

c Draw radii OA and OB. Find the area of triangle AOB. [3]

32 Acute triangle ABC is inscribed in circle O. The radius of the circle is 10 inches, and sides AB and BC are each 16 inches. OE is drawn perpendicular to side AB, meeting AB at E. OA, OB and OC are drawn.

a Find to the *nearest degree* angle EOB. [3]

b Using the result found in answer to *a*, find angle AOB and obtuse angle AOC. [1, 3]

c Using a result found in answer to *b*, find to the *nearest inch* the distance from O to chord AC. [3]

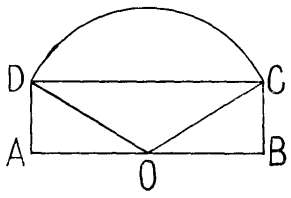
33 In trapezoid $ABCD$, base AB is 8 inches and base DC is 20 inches. Sides DA and CB are extended to meet at point G. The altitude of the trapezoid is 3 inches longer than the altitude to side AB of triangle GAB.

a Using x to represent the altitude from G in triangle GAB, represent the corresponding altitude in triangle GDC. [2]

b Find the length of the altitude of the trapezoid. [6]

c Find the area of the trapezoid. [2]

34 In the accompanying figure, $ABCD$ is a rectangle. O is the mid-point of the longer side AB and is the center of the arc passing through points D and C. If AD equals 4 feet and angle DOC equals 120°, find the area of the entire figure to the *nearest square foot*. [10]



Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The vertex angle of an isosceles triangle contains 70°. Find the number of degrees in a base angle. 1.....

2 Two angles are supplementary, and one is 32° larger than the other. Find the number of degrees in the smaller angle. 2.....

3 Find the number of degrees in an angle of a regular pentagon. 3.....

4 Find the radius of a circle whose area is 36π. 4.....

5 Find the area of an equilateral triangle whose side is 8. [Answer may be left in radical form.] 5.....

6 The hypotenuse of a right triangle is 12 and one leg is 6. Find the shorter segment of the hypotenuse made by the altitude upon the hypotenuse. 6.....

7 A secant and a tangent are drawn to a circle from an external point. If the secant is 16 and its external segment is 4, find the tangent. 7.....

8 A central angle of a circle whose radius is 18 contains 50°. Find the length of its arc. [Answer may be left in terms of π.] 8.....

9 In triangle ABC, angle C equals 90°, angle A equals 71° and AC equals 20 inches. Find to the *nearest inch* the length of BC. 9.....

10 One side of a polygon is 2 and the corresponding side of a similar polygon is 3. Find the ratio of the areas of the two polygons. 10.....

11 The area of a rhombus is 72. If one diagonal is 16, find the other diagonal. 11.....

12 Find the length of the line segment which joins the mid-points of two sides of a triangle whose third side is 10 inches. 12.....

13 The perimeters of two regular polygons having the same number of sides are 12 and 18. Find the ratio of the corresponding sides. 13.....

14 Two parallelograms have the same base. If their altitudes are in the ratio 4 : 5, what is the ratio of their areas? 14.....

15 The legs of a right triangle are 3 and 4. One leg of another right triangle is 4 and its hypotenuse is 5. Are the triangles congruent? [Answer *yes* or *no*.] 15.....

16 In parallelogram $ABCD$, angle A is a right angle. Is diagonal AC equal to diagonal BD? [Answer *yes* or *no*.] 16.....

[OVER]

Directions (17–23): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

17 A circle can always be circumscribed about any (*a*) parallelogram (*b*) rhombus (*c*) rectangle 17.....

18 Chords AB and CD of a circle intersect in E. If arc AD is $4m°$ and arc BC is $2m°$, then angle AED contains (*a*) $m°$ (*b*) $2m°$ (*c*) $3m°$ 18.....

19 Point C is 3 inches from given line AB. The number of points in AB 5 inches from C is (*a*) 0 (*b*) 1 (*c*) 2 19.....

20 In triangle ABC, angle C equals 60° and AB is greater than AC. Angle B is (*a*) equal to 60° (*b*) less than 60° (*c*) greater than 60° 20.....

21 As the number of sides of a polygon increases, the number of degrees in the sum of the exterior angles (*a*) increases (*b*) decreases (*c*) remains the same 21.....

22 If A, B, C and D are four consecutive points on a circle such that arc AB equals arc CD, then chords BC and AD always (*a*) are equal (*b*) intersect (*c*) are parallel 22.....

23 Two unequal circles are tangent externally. From a point on their common internal tangent, tangents are drawn to the two circles. The tangent to the larger circle is (*a*) greater than the tangent to the smaller circle (*b*) equal to the tangent to the smaller circle (*c*) less than the tangent to the smaller circle 23.....

Directions (24–25): Leave all construction lines on your paper.

24 Divide line segment AB into three equal parts.

A ——————————— B

25 Locate a point in side BC that is equidistant from AB and AC.