The University of the State of New York

315TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Monday, June 16, 1952 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. **At the end of that time, this part of the examination must be detached and will be collected by the teacher.** If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove that the sum of the interior angles of a triangle is a straight angle. [10]

27 BC is the base of isosceles triangle ABC inscribed in a circle. P is any point on BC. AP is drawn and extended to meet the circle in S, and chord SC is drawn.

Prove: (1) $\angle ACP = \angle ASC$ [5]
         (2) $AP : AC = AC : AS$ [5]

28 Prove that if in the same or equal circles, two chords are equidistant from the center, the chords are equal. [10]

29 In an acute triangle ABC, the altitude from B meets side AC in D. If angle ABD is greater than angle CBD, prove that AB is greater than BC. [10]

Part III

Answer two questions from part III.

30 BA and BC are tangents to circle O at A and C respectively and form an angle of $50°$. Lines OA, OB and OC are drawn. The length of OB is 14.2 inches.

   a Find to the *nearest inch* the length of the radius of the circle. [6]
   b Using the value of the radius obtained in answer to *a* and using $\pi = \frac{22}{7}$, find to the *nearest square inch* the area of minor sector AOC. [4]
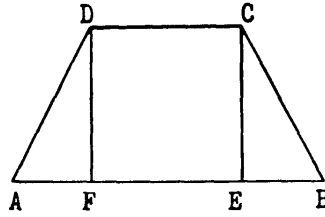
[1]   [OVER]

31 In the accompanying diagram, $ABCD$ is an isosceles trapezoid, $\angle B = 60°$, and DF and CF are altitudes.

a If EB is represented by x, represent CB in terms of x. [2]

b If DC exceeds CB by 2, represent the perimeter of the trapezoid in terms of x. [2]

c If the perimeter of the trapezoid is 44, find x. [2]

d Find the area of the trapezoid. [Answer may be left in radical form.] [4]

32 In parallelogram $ABCD$, diagonals AC and BD intersect in O, and AK is perpendicular to BD. The area of $ABCD = 336$, $AC = 30$ and $BD = 28$.

a Find the area of triangle ABD. [1]

b Find the length of AK; of OK. [3, 3]

c Find the length of AD. [3]

[2]

Fill in the following lines:

Name of pupil..Name of school..

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The sides of a rectangle are 12 and 16. Find the diagonal.

1.................

2 Find the area of a circle whose radius is 6. [Answer may be left in terms of π.]

2.................

3 Find the length of an arc of 45° in a circle whose radius is 8. [Answer may be left in terms of π.]

3.................

4 The angles of a triangle are in the ratio 1:3:5. Find the number of degrees in the smallest angle of the triangle.

4.................

5 The perimeter of a regular polygon is 22 and its apothem, to the nearest integer, is 3. Find the area of the polygon.

5.................

6 From a point outside a circle a tangent and a secant are drawn. If the tangent is 8 and the secant is 16, find the external segment of the secant.

6.................

7 Chord AB of a circle has an arc of 50°. Find the number of degrees in the acute angle formed by AB and the tangent to the circle at A.

7.................

8 Chords AB and CD intersect within a circle at point E. If arc $AC = 40°$ and arc $DB = 50°$ find the number of degrees in angle AEC.

8.................

9 In a circle, chord DC bisects chord AB at E. If $DE = 12$ and $EC = 3$, find AE.

9.................

10 The line segment joining the mid-points of two sides of equilateral triangle ABC is 6. Find the perimeter of triangle ABC.

10.................

11 Find the area of a rhombus whose diagonals are 12 and 5.

11.................

12 In right triangle ABC, CD is the altitude to the hypotenuse. If $AD = 6$ and $AB = 10$, find AC. [Answer may be left in radical form.]

12.................

13 Find to the *nearest degree,* the angle of elevation of the sun when a vertical 10-foot pole casts a shadow 4 feet long.

13.................

14 Find the area of an equilateral triangle whose side is 10. [Answer may be left in radical form.]

14.................

[3]

15 In triangle ABC, angle C is 60° and angle A is 65°. Name the shortest side of the triangle. 15................

16 If corresponding sides of two similar polygons are in the ratio 1:3, find the ratio of their areas. 16................

17 Is statement A the converse of statement B? [Answer *yes* or *no*.]
    A Two triangles are similar if their corresponding angles are equal.
    B The corresponding angles of two similar triangles are equal. 17................

18 How many points are there that are equally distant from two given points A and B and also 1 inch from the straight line passing through A and B? 18................

19 Can more than three angles of a pentagon be right angles? [Answer *yes* or *no*.] 19................

Directions (20–23): Indicate the correct completion for *each* of the following by writing on the line at the right the letter a, b or c.

20 The median to the hypotenuse of a right triangle divides the triangle into two triangles that are always   (a)right triangles   (b)isosceles triangles (c)congruent triangles 20......

21 The diagonals of a quadrilateral divide it into four congruent triangles if the quadrilateral is   (a)a rectangle   (b)a rhombus   (c)an isosceles trapezoid 21......

22 A triangle is equal in area to a parallelogram. If the base of the triangle is equal to the base of the parallelogram, the altitude of the triangle is   (a)equal to the altitude of the parallelogram   (b)one half the altitude of the parallelogram (c)twice the altitude of the parallelogram 22......

23 If triangles are constructed on a given line segment as base and the altitude to the base is a given length, then the locus of the vertex opposite the base is (a)a circle   (b)a straight line   (c)two straight lines 23......
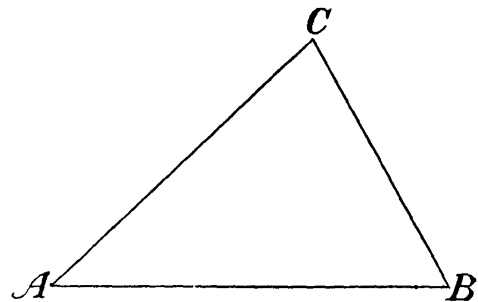
Directions (24–25): Leave all construction lines on your paper.

P.

24 Through P construct a line parallel to line AB.
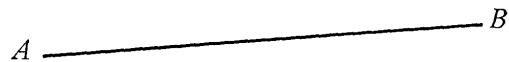
25 Find by construction the center of the circle that can be inscribed in triangle ABC.

[4]