The University of the State of New York

306TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Monday, June 20, 1949 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove that if two sides of a quadrilateral are equal and parallel, the figure is a parallelogram. [10]

27 Secants PAB and PCD are drawn to a circle from an external point P and chords AB and CD thus formed are equal. Chord BD is drawn. Prove that

a arc AB = arc DC [1]

b $\angle B = \angle D$ [4]

c $AP = CP$ [5]

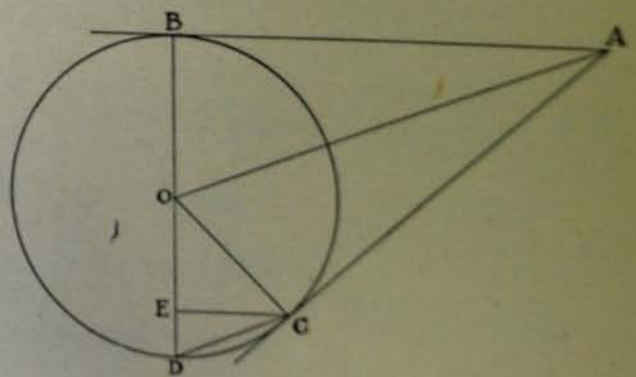
28 Prove that the area of a triangle is equal to one half the product of its base and its altitude. [10]

29 AB and AC are tangents to circle O at points B and C respectively. From C a line is drawn perpendicular to diameter BD and intersecting BD at E. Lines AO, OC and DC are drawn. Prove that

a AO bisects arc BC [4]

b $\angle BOA = \angle EDC$ [3]

c $AB : CE = BO : ED$ [3]



Part III

Answer one question from part III.

30 Angle B of isosceles triangle ABC is 126° and side AB is 5 inches.

a Find, to the *nearest tenth of an inch*, the length of altitude BD. [5]

b Find, to the *nearest tenth of an inch*, the length of AD. [3]

c Find, to the *nearest square inch*, the area of triangle ABC. [2]

31 The diagonals of a rhombus are in the ratio 5 : 12.

a If the shorter diagonal is represented by $5x$, represent the longer diagonal in terms of x. [1]

b Express the area of the rhombus in terms of x. [2]

c If the area of this rhombus is 120 square inches, find the length of each diagonal. [4]

d Find one side of the rhombus. [3]

Part IV

Answer one question from part IV.

32 For *each* of the following statements indicate whether you are given *too little* information, *just enough* information, or *more* information than is needed, to justify the conclusion.

a If a quadrilateral is inscribed in a circle, the sum of its angles is 360°. [2]

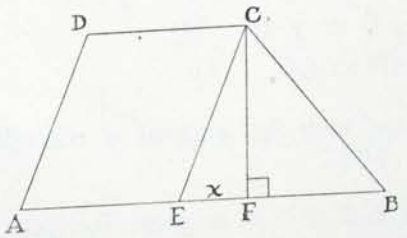
b If a diagonal of a quadrilateral divides it into two congruent triangles, the quadrilateral is a parallelogram. [2]

c If a quadrilateral is circumscribed about a circle, the sum of two opposite sides equals the sum of the other two sides. [2]

d The line joining the mid-points of the legs of a trapezoid bisects each diagonal. [2]

e If a quadrilateral is a parallelogram, the figure formed by joining in order the mid-points of its sides is a parallelogram. [2]

33 The bases AB and CD of a trapezoid are 24 and 10 and the legs AD and BC are 13 and 15 respectively. CE is drawn parallel to DA, and CF is perpendicular to AB as shown in the figure. Let EF be represented by x.



a Express FB in terms of x. [2]

b Using triangles EFC and BFC, write *two* expressions for $(CF)^2$ in terms of x. [2, 2]

c Find the value of x. [2]

d Find the area of the trapezoid. [2]

[2]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Two chords intersecting within a circle intercept opposite arcs of 40° and 80°. Find the number of degrees in the acute angle formed by the chords. 1.....

2 A secant and a tangent to a circle from an external point are 8 and 4 respectively. Find the external segment of the secant. 2.....

3 The median to the hypotenuse of a right triangle is 5. Find the hypotenuse. 3.....

4 A side of a triangle is 12. Find the length of the line segment joining the mid-points of the other two sides. 4.....

5 A square is equal in area to a parallelogram whose base is 8 and whose altitude is 2. Find a side of the square. 5.....

6 Find the altitude of an equilateral triangle whose side is 5. [Answer may be left in radical form.] 6.....

7 A line parallel to side AB of triangle ABC intersects AC at D and BC at E. If $DC = 12$, $AD = 4$ and $EC = 18$, find BE. 7.....

8 Corresponding sides of two similar polygons are in the ratio 3:4. Find the ratio of their areas. 8.....

9 An exterior angle of a regular polygon is 40°. Find the number of sides of the polygon. 9.....

10 Find the circumference of a circle whose radius is 14. [Use $\pi = \frac{22}{7}$] 10.....

11 The angle of a sector of a circle is 60° and the radius of the circle is 6. Find the area of the sector. [Answer may be left in terms of π.] 11.....

12 CD is the altitude on the hypotenuse of right triangle ABC. $AD = 2$ and $DB = 8$. Find CD. 12.....

13 AB and CD are two parallel lines 4 inches apart and P is a point on AB. How many points are there which are equidistant from AB and CD and 3 inches from P? 13.....

14 Find the diagonal of a rectangle whose sides are 8 and 15. 14.....

15 In parallelogram $ABCD$, angle $A = 30°$. How many degrees are there in angle B? 15.....

16 The sides of a rectangle are 8 and 10. Find, to the *nearest degree*, the angle formed by the diagonal and the longer side of the rectangle. 16.....

Directions (questions 17–23) — If the blank in each statement is replaced by one of the words *always, sometimes,* or *never,* the resulting statement is true. Select the word that will correctly complete *each* statement and write the word on the line at the right.

17 The diagonals AC and BD of quadrilateral $ABCD$ inscribed in a circle intersect at E. Triangle AED is ... similar to triangle BEC. 17.....

18 From external point A tangents AB and AC are drawn to a circle and chord BC is drawn. Triangle ABC is ... equilateral. 18.....

19 The locus of the center of a circle of given radius and tangent externally to a given circle is ... a circle. 19.....

[OVER]

20 The areas of two triangles having equal altitudes ... are to each other as their bases. 20.....

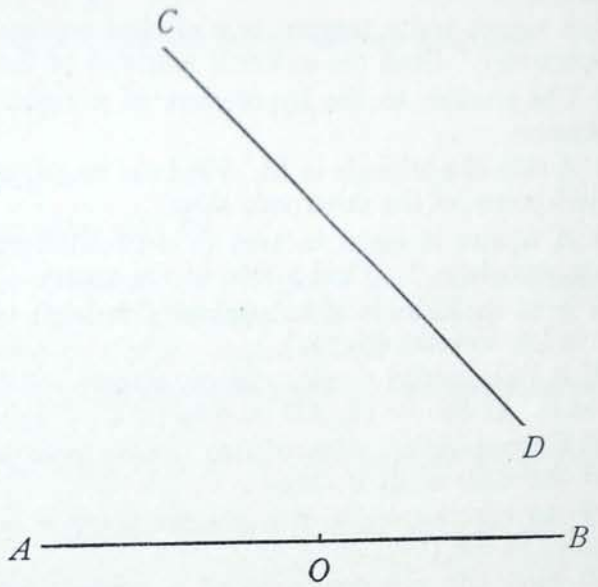
21 The medians of a triangle ... bisect each other. 21.....

22 If angle A of triangle ABC is greater than angle A' of triangle $A'B'C'$, then BC is ... greater than $B'C'$. 22.....

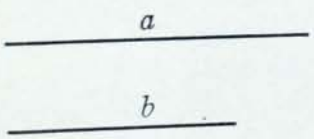
23 It is ... possible to construct triangle ABC if the given parts are side AB, angle A and the altitude on AB. 23.....

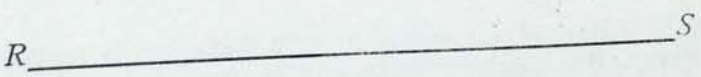
Directions (questions 24–25) — Leave all construction lines on the paper.

24 Construct a circle which is tangent to line AB at point O and whose center is on line CD.



25 Divide line segment RS into segments having the ratio $a:b$.





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