The University of the State of New York

300TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, June 17, 1947 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove that if the opposite sides of a quadrilateral are equal, the figure is a parallelogram. [10]

27 Prove that if in a right triangle the perpendicular is drawn from the vertex of the right angle to the hypotenuse,

a The two triangles thus formed are similar to each other [6]

b The perpendicular is the mean proportional between the segments of the hypotenuse [4]

28 The bisector of angle C of the inscribed triangle ABC intersects AB in F and the circle in K. Prove: $AC : FC = KC : BC$ [10]

29 The bisector of angle B of triangle ABC intersects AC in D. Prove that AB is greater than AD. [10]

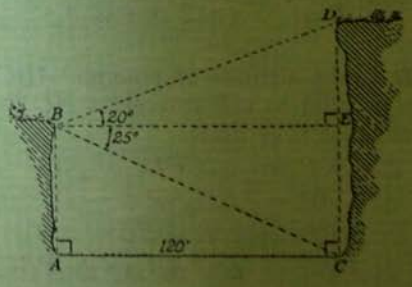
Part III

Answer one question from part III.

30 AB and CD are cliffs on opposite sides of a river 120 feet wide. From B the angle of elevation of D is 20° and the angle of depression of C is 25°. Find, correct to the *nearest foot*,

a The height of cliff AB [5]

b The height of cliff CD [5]

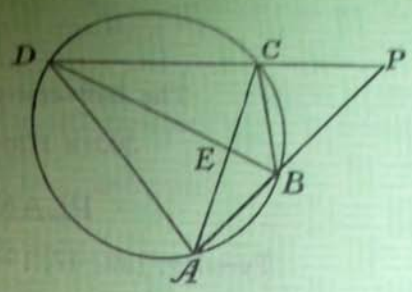


31 In the drawing at the right, arcs AB and BC are equal; arc CD is twice arc BC; arc DA is 30° more than arc CD.

a If x represents the number of degrees in arc AB, write in terms of x the number of degrees in arcs BC, CD and DA. [2]

b Form an equation in x and find the number of degrees in arcs BC and DA. [3]

c Find the number of degrees in angles AED, APD and BCP. [1, 1, 3]



Part IV

Answer one question from part IV.

32 Each of the five parts of this question is a statement that may be correctly completed by *one or more* of the given choices. Write the numbers (1) to (5) on your answer paper and after *each* indicate the correct answer or answers to the corresponding question by writing one or more of the letters *a*, *b*, *c*, *d*. [10]

[In each of the five parts of the question, one credit will be allowed for each correct choice made and one credit will be deducted for each incorrect choice. The minimum credit on each part will be 0.]

(1) In any right triangle (*a*) the perpendicular bisectors of the legs meet on the hypotenuse (*b*) the area is one half the product of the legs (*c*) the altitudes meet inside the triangle (*d*) the median to the hypotenuse bisects the right angle

(2) In any isosceles triangle (*a*) the base is shorter than either leg (*b*) two altitudes are equal (*c*) each exterior angle at the base is obtuse (*d*) any point on the median to the base is equidistant from the ends of the base

(3) In any parallelogram (*a*) the diagonals are equal (*b*) a circle may be inscribed (*c*) the opposite angles are equal (*d*) the area is the product of two adjacent sides

(4) In any regular pentagon in which r is the apothem, s the side and R the radius of the circumscribed circle

(*a*) $\frac{s}{2} = r \tan 36°$ (*b*) $R = r \sin 54°$

(*c*) area of the pentagon $= \frac{5rs}{2}$ (*d*) $R = \sqrt{r^2 + s^2}$

(5) Given point P on line l. The number of possible points 4″ from l and r distance from P is (*a*) 2 if r is 3″ (*b*) 4 if r is 5″ (*c*) 1 if r is 4″ (*d*) 0 if r is 1″

33 CD is the altitude of triangle ABC and E is a point on CD such that $DE = \frac{1}{3} CD$. Through E a line parallel to AB intersects AC in K and BC in L. P, the midpoint of KL, is joined to A, B and C. Prove that:

a $\triangle CKP = \triangle CPL$ [2]

b $\triangle CAP = \triangle CBP$ [4]

c $\triangle ABP = \frac{1}{3} \triangle ABC$ [2]

d $\triangle PAC = \triangle PAB = \triangle PBC$ [2]

Fill in the following lines:

Name of school..... Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 One angle of a right triangle is 30° and the leg opposite this angle is 4. Find the hypotenuse. 1.....

2 Find the altitude of the equilateral triangle whose side is 8. [Answer may be left in radical form.] 2.....

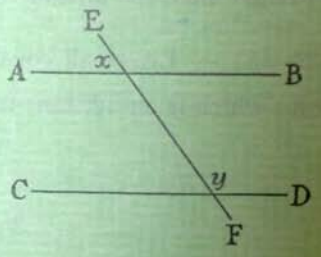
3 The side of a square is 8. Find the length of the diagonal. [Answer may be left in radical form.] 3.....

4 How many degrees are there in the sum of the exterior angles of any polygon, if one angle is taken at each vertex? 4.....

5 A circle is divided into 6 equal arcs. If the diameter of the circle is 12 inches, how long is the chord joining any two successive points of division? 5.....

6 Quadrilateral $ABCD$ is inscribed in a circle. If angle A equals 75°, find the number of degrees in angle C. 6.....

7 Parallel lines AB and CD are cut by transversal EF, making angle $x = 55°$. Find the number of degrees in angle y.



7.....

8 In triangle RST a line is drawn parallel to ST, intersecting RS in K and RT in L. If $RK = 5$, $KS = 10$ and $RT = 27$, find the length of RL. 8.....

9 One of the base angles of an isosceles trapezoid is 40° and one of the equal sides is 5 feet. Find the altitude of the trapezoid correct to the *nearest foot*. 9.....

10 Find the area of a trapezoid whose bases are 12 and 7 and whose altitude is 8. 10.....

11 If the side of an equilateral triangle is 5, find the area of the triangle. [Answer may be left in radical form.] 11.....

12 The diagonals of a rhombus are 5 and 12. Find the area of the rhombus. 12.....

13 The areas of two similar triangles are in the ratio 1 : 4. Find the ratio of a pair of corresponding sides. 13.....

14 If the area of a circle is 36π, find its circumference. [Answer may be left in terms of π.] 14.....

15 In circle O chord CD bisects chord AB at E. If $CE = 3$ and $ED = 12$, find the length of AE. 15.....

16 From a point A outside circle O, tangent AB and secant ACD are drawn. If AB is 6 and the external segment AC of the secant is 4, find the length of the entire secant. 16.....

[OVER]

17 What is the locus of points outside circle O and at a distance m from the circle? 17.....

18 Is statement b a converse of statement a? [Answer *yes* or *no*.]

a Parallel chords intercept equal arcs on a circle.

b Two chords intercepting equal arcs on a circle are parallel. 18.....

19 Below are given four terms arranged in different orders. Which of these, *a*, *b*, *c* or *d*, represents the sequence in which the definitions of these four terms should be given?

a Polygon, quadrilateral, rectangle, parallelogram

b Polygon, rectangle, parallelogram, quadrilateral

c Polygon, quadrilateral, parallelogram, rectangle

d Polygon, parallelogram, rectangle, quadrilateral 19.....

Directions (questions 20–23) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement is true. Select the word that will correctly complete *each* statement and write the word on the line at the right.

20 A line perpendicular to a chord is ... a diameter. 20.....

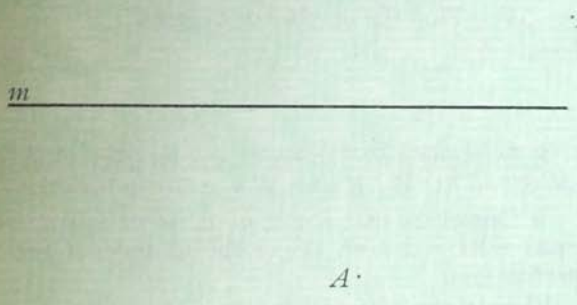
21 A median ... divides a triangle into two congruent triangles. 21.....

22 A trapezoid inscribed in a circle is ... isosceles. 22.....

23 If all the sides of a triangle are unequal, an angle bisector terminating in the opposite side is ... greater than the altitude drawn to that side. 23.....

Directions (questions 24–25) — Leave all construction lines on the paper.

24 On line m find the point which is equidistant from points A and B.



25 Given triangle ABC, construct on base DE, corresponding to AC, a triangle similar to triangle ABC.