PLANE GEOMETRY

Tuesday, June 18, 1946 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove that the diagonals of a parallelogram bisect each other.   [10]

27 In triangle ABC, the bisectors of angles A and B meet in D. Through D a line is drawn parallel to AB, intersecting AC at E and BC at F.
 a Prove that triangle ADE and triangle BDF are isosceles.   [7]
 b Prove: $EF = AE + BF$ [3]
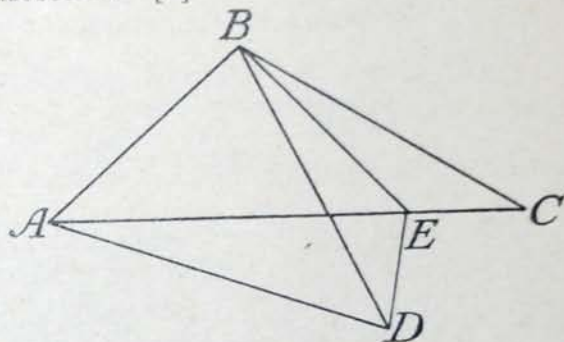
28 In the figure at the right, $BC = BD$ and BE is the bisector of angle DBC.
 a Prove: $ED = EC$ [5]
 b Using the fact that $ED = EC$, prove: $AC > AD$ [5]

29 Prove that the area of a triangle is equal to one half the product of the base and altitude.   [10]

Part III

Answer one question from part III.

30 A side of a regular polygon of 10 sides is 8.
 a Find, correct to the *nearest tenth*, the apothem of the polygon.   [6]
 b Using the result obtained in answer to *a*, find the area of the polygon.   [4]

31 In the figure at the right, AB is tangent to circle O at point B, AEC is a secant, and DE, FC and BC are chords. Arc $EB = 60°$, arc $BC = 140°$, arc $CD = 60°$, arc $DF = 55°$. Find the number of degrees in arc FE, angle a, angle b, angle c and angle d.   [10]

[1]   [OVER]
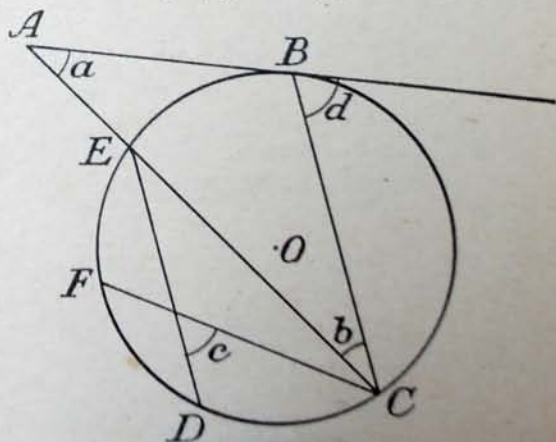
Part IV
Answer one question from part IV.

32 Each of the following statements may be correctly completed by *one or more* of the given choices. Write the numbers (1)–(5) on your answer paper and after *each* indicate the correct answer to the corresponding question by writing one or more of the letters *a, b, c, d.* [10]

[In each of the five parts of this question, one credit will be allowed for each correct choice made and one credit will be deducted for each incorrect choice. The minimum credit on each part will be 0.]

(1) A quadrilateral is a parallelogram if (*a*) two of its sides are equal and parallel, (*b*) its diagonals are equal, (*c*) it is inscribed in a circle, (*d*) its opposite angles are equal

(2) Two triangles are always congruent if the corresponding parts which are given equal are (*a*) the three sides, (*b*) any two sides and an angle, (*c*) any two angles and a side, (*d*) the three angles

(3) The areas of any two regular polygons of the same number of sides are to each other as (*a*) their perimeters, (*b*) the squares of their apothems, (*c*) the product of their perimeters and apothems, (*d*) the squares of the radii of their circumscribed circles

(4) Two polygons are always similar if (*a*) they are similar to the same polygon, (*b*) the sides of one are proportional to the sides of the other, (*c*) the angles of one are equal to the corresponding angles of the other, (*d*) they are composed of the same number of triangles similar each to each and similarly placed

(5) Given two intersecting lines *m* and *n*, the point *P* and the distance *s;* the locus of points equally distant from *m* and *n* and also at a distance *s* from *P* (*a*) always consists of four points, (*b*) always consists of two points, (*c*) may be only one point, (*d*) may consist of more than four points

33 A belt is placed over unequal wheels, *A* and *B*, as shown by the heavy line in the figure at the right. Wheel *A* has an 8-inch radius and wheel *B* has a 2-inch radius. The distance *AB* is 12 inches.

a Find the number of degrees in angle *DAB*. [Suggestion: *GB* is drawn parallel to *CD*] [1]

b Find, correct to the *nearest tenth of an inch,* the length of the section of the belt represented by the common external tangent *CD*. [2]

c Find, correct to the *nearest tenth of an inch,* the lengths of the sections of the belt represented by arcs *DHE* and *CKF*. [3, 3]

d Find, correct to the *nearest inch,* the length of the entire belt. [1]

[2]
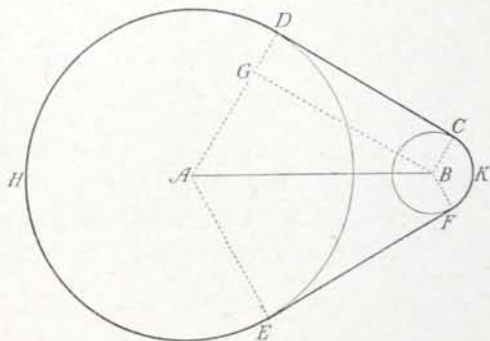
Fill in the following lines:

Name of pupil..Name of school..

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 The sides of a triangle are 3, 4 and 5. Find the area of the triangle.   1.....................

2 Find the area of a circle whose radius is 6. [Answer may be left in terms of π.]   2.....................

3 Find the area of a rhombus whose diagonals are 8 and 6.   3.....................

4 The area of a trapezoid is 60 square inches. The altitude is 4 inches and one base is 20 inches. Find the number of inches in the length of the other base.   4.....................

5 The sides of a parallelogram are 6 and 8 and the included angle is 30°. Find the altitude of the parallelogram on side 8.   5.....................

6 The angle formed by two tangents is 40°. Find the number of degrees in the smaller intercepted arc.   6.....................

7 Two parallel lines are cut by a transversal. The number of degrees in the two interior angles on the same side of the transversal are represented algebraically by x and $4x + 30$. How many degrees are there in the smaller of the two angles?   7.....................

8 How many degrees are there in each interior angle of a regular pentagon?   8.....................

9 Find the area of an equilateral triangle whose side is 4. [Answer may be left in radical form.]   9.....................

10 The sides of a triangle are 8, 15 and 17. Find the tangent of the smaller acute angle.   10.....................

11 Find the diagonal of a square whose side is 8. [Answer may be left in radical form.]   11.....................

12 The altitude on the hypotenuse of a right triangle divides the hypotenuse into segments 3 and 12. Find the altitude.   12.....................

13 In triangle ABC, a line parallel to AC cuts AB at D and BC at E. If $AC = 8$, $DE = 6$ and $AB = 12$, find BD.   13.....................

14 In a circle, the chords AB and CD intersect at E. If $AE = 3$, $EB = 4$ and $CE = 2$, find ED.   14.....................

15 In right triangle ABC, angle $A = 34°$, angle $C = 90°$ and $AB = 20$. Find BC correct to the *nearest tenth*.   15.....................

Directions (questions 16–19) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

16 If two circles are tangent internally, they have (*a*) no common tangents, (*b*) one common tangent, (*c*) more than one common tangent   16......

17 Two isosceles triangles with equal vertex angles are always (*a*) congruent, (*b*) similar, (*c*) equal in area   17......

18 The number of points which are at a given distance from a given line and also equally distant from two given points on the line is (*a*) one, (*b*) two, (*c*) four   18......

19 If the point of intersection of the perpendicular bisectors of the sides of a triangle is outside the triangle, the triangle is (*a*) acute, (*b*) right, (*c*) obtuse   19......

[3]

Directions (questions 20–23) — If the blank in each statement is replaced by one of the words *always, sometimes* or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

20 If the diagonals of a parallelogram are equal, the parallelogram is ... a square.

20................

21 If a diameter of a circle intersects a chord, it is ... perpendicular to the chord.

21................

22 An exterior angle of a right triangle can ... be an acute angle.

22................

23 Sectors having angles of the same number of degrees and drawn in unequal circles are ... equal.

23................

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Construct the median to side *AC* of triangle *ABC*.
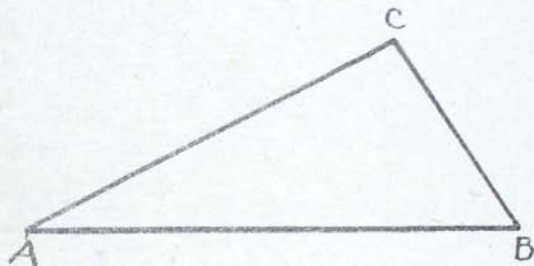
25 Find by construction one side of an equilateral triangle whose perimeter is equal to the given line segment *AB*.