# PLANE GEOMETRY

Tuesday, June 15, 1943 — 9.15 a. m. to 12.15 p. m., only

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## Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

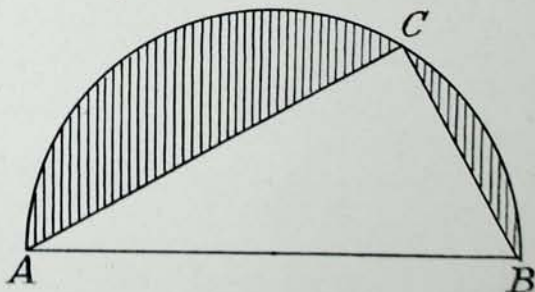## Part II
### Answer two questions from part II.

26 Prove that in an isosceles triangle the angles opposite the equal sides are equal.   [10]

27 On diameter $AB$ of semicircle $O$ two points, $C$ and $D$, are located on opposite sides of the center $O$ so that $AC = BD$. At $C$ and $D$ perpendiculars are erected to $AB$ and extended to meet the semicircle in points $E$ and $F$ respectively. Prove that $CE$ equals $DF$.   [10]

28 Prove that two triangles are similar if an angle of one triangle is equal to an angle of the other and the sides including the angles are in proportion.   [10]

## Part III
### Answer two questions from part III.

29 Given a circle whose radius is 4; a chord of this circle is drawn perpendicular to and bisecting one radius.

    *a* How many degrees are there in the minor arc of this chord?   [4]

    *b* Find the length of this arc to the *nearest integer*.   [Use $\pi = 3.14$]   [3]

    *c* Find the length of the chord correct to the *nearest integer*.   [3]

30 The diagonals of a rectangle are each 22 and intersect at an angle of 110°. Find, correct to the *nearest integer*, the sides of the rectangle.   [Use numerical trigonometry.]   [6, 4]

31 In the semicircle at the right, chord $AC = 8$ and chord $BC = 6$. Find the area of the shaded portion correct to the *nearest integer*.   [Use $\pi = 3.14$]   [10]

## Part IV
### Answer one question from part IV.

**32** *AB* is a given line segment which is fixed in position. Triangles are drawn having *AB* as one side and *C* as the vertex opposite side *AB*. Describe the locus of vertex *C* if

    *a* Triangle *ABC* is isosceles   [2]

    *b* Angle *C* is a right angle   [2]

    *c* The median upon *AB* is a given line segment   [3]

    *d* Triangle *ABC* has a constant area   [3]

**33** *a* Given:  *PAB* is a secant of circle *O* and *PD* is a line drawn to circle *O* such that

    $(PD)^2 = PA \times PB$

  Prove:  *PD* is tangent to circle *O*.

  Suggestions:  Draw *PE* tangent to circle *O* at *E*; also draw radii *OE* and *OD* and line *OP*.  Then prove

    (1) $PE = PD$   [3]

    (2) $\triangle POD \cong \triangle POE$   [2]

    (3) *PD* is tangent to circle *O*   [4]
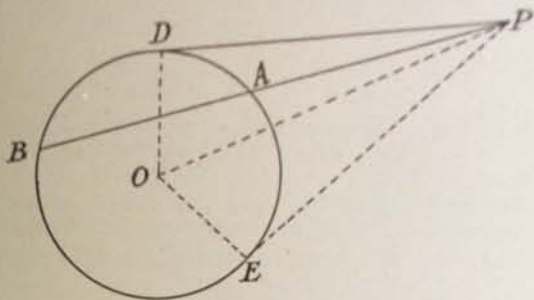
  *b* State the proposition of which this exercise is a converse.   [1]

[2]

## Fill in the following lines:

Name of school................................................Name of pupil........................................................

## Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–3) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

1 The sum of the interior angles of a polygon is 1800 degrees. The polygon has
(*a*) 8 sides, (*b*) 10 sides, (*c*) 12 sides

2 If two altitudes of a given triangle fall outside the triangle, the triangle is
(*a*) right, (*b*) acute, (*c*) obtuse

3 If the length of the apothem of a regular pentagon is *m*, then the length of half of one side of the pentagon is (*a*) $m \cos 36°$, (*b*) $m \sin 36°$, (*c*) $m \tan 36°$

1......

2......

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Directions (questions 4–9) — If the blank in each statement is replaced by one of the words *always, sometimes* or *never*, the resulting statement is true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

4 A parallelogram whose diagonals are equal and perpendicular is ... a square.

5 The perpendicular bisector of a chord of a circle ... passes through the center of the circle.

6 If two isosceles triangles have a side of one equal to the corresponding side of the other, the triangles are ... congruent.

7 If the sides of one triangle are parallel to the sides of another triangle, the triangles are ... similar.

8 All three exterior angles of a triangle are ... acute.

9 If two sides of a quadrilateral are parallel and the other two sides are equal, the quadrilateral is ... a parallelogram.

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Directions (questions 10–23) — Write the answer to *each* question on the line at the right.

10 The bases of a trapezoid are 8 and 12 and its area is 140. What is its altitude?

11 The three sides of a triangle are 8", 10" and 12". What is the length of the line segment drawn from the mid-point of the 8" side to the mid-point of the 12" side?

12 A vertical pole 10' high casts a shadow 8' long and at the same time a near-by tree casts a shadow 40' long. What is the height of the tree?

13 The sides of an angle, formed by a tangent and a secant drawn to a circle from an external point, intercept arcs of 140° and 30°. How many degrees are there in the angle?

14 Quadrilateral *ABCD* is inscribed in a circle. If angle *A* contains 100 degrees, find the number of degrees in angle *C*.

15 If the radius of a circle is doubled, by what number is the circumference of the circle multiplied?

16 A regular hexagon is inscribed in a circle and a tangent is drawn to the circle at a vertex of the hexagon. How many degrees are there in one of the acute angles thus formed?

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[3]

[OVER]

17 One acute angle of a right triangle is double the other. The longer leg is $5\sqrt{3}$. How long is the hypotenuse?

18 An arc of a circle contains 72° and is 10 inches long. Find the circumference of the circle.

19 The area of the sector of a circle whose angle is 40° is $4\pi$. Find the radius of the circle.

20 If two polygons are similar and a side of one is two thirds as long as the corresponding side of the other, what is the ratio of the area of the smaller polygon to the area of the larger polygon?

21 The legs of an isosceles triangle are $AC$ and $AB$, and $AD$ is a median. $AC = 5$ and $AD = 4$. Find the length of the base $BC$.

22 How many points are there which are 2 inches from a given line and 3 inches from a given point in that line?

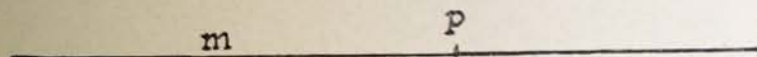23 Triangle $ABC$ is a right triangle with right angle at $C$ and $E$ is any point on side $AC$. Line $ED$ is perpendicular to $AB$ at $D$. Complete the proportion $DE : BC = DA : \ldots$

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Directions (questions 24–25) — Leave all construction lines on the paper.

24 At point $P$ construct a line making an angle of 30° with line $m$.

_____ m _____ P _____

25 Divide line segment $a$ into *three* equal parts.

_____ a _____