The University of the State of New York

284TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, June 16, 1942 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II, III and IV

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

See instructions for parts II, III and IV on page 1.

Part II

Answer two questions from this part.

26 Prove that an angle formed by two chords intersecting within a circle is measured by one half the sum of the intercepted arcs. [10]

27 Prove that the diagonals of a trapezoid divide each other proportionally. [10]

28 *a* Prove that any point which is equidistant from the sides of an angle is on the bisector of the angle. [7]

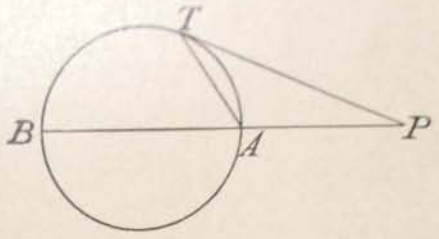
b Write the converse of the proposition stated in *a*. [2]

c Is either the proposition in *a* or its converse, taken alone, sufficient to warrant the conclusion that the locus of points equidistant from the sides of an angle is the bisector of the angle? [1]

Part III

Answer two questions from this part.

29 In the accompanying figure, PT is a tangent, PAB is a secant and TA a chord. Arc $BT = 112°$ and angle $T = 34°$.



a Find the number of degrees in angle BAT. [2]

b Find the number of degrees in angle P. [4]

c Show that AB is a diameter of the circle. [4]

30 In a given circle the line segment joining the mid-point of a chord and the mid-point of its minor arc is 4 inches long. The chord is 16 inches long.

a Find the radius of the circle. [Show all work.] [7]

b Find the area of the triangle formed by the chord and the radii drawn to its end points. [3]

31 A regular pentagon whose side is 22.0 inches is inscribed in a circle.

a Find, correct to the *nearest tenth of an inch*, the apothem of the pentagon. [6]

b Using the result obtained in answer to *a*, find, correct to the *nearest square inch*, the area of the pentagon. [4]

Part IV

Answer one question from this part.

32 In each of the following propositions the hypothesis contains one or more facts which we do not need to know in order to prove the conclusion. In *each* case, merely write the fact or facts *not needed*.

a If one base of a trapezoid is twice the other and the diagonals are equal, then the trapezoid is isosceles. [2]

b If a leg and the adjacent acute angle of one right triangle are equal to the corresponding parts of another right triangle, then the triangles are congruent. [2]

c In a quadrilateral having two sides equal and parallel, the line segments joining the mid-points of the four sides, taken in order, form a parallelogram. [2]

d The areas of two similar triangles are to each other as the product of the base and the altitude of one is to the product of the base and the altitude of the other. [2]

e If the sum of the exterior angles of a regular polygon made by producing each of its sides in succession is one half the sum of its interior angles, then the polygon has six sides. [2]

33 $ABCD$ is a parallelogram whose diagonals AC and BD intersect in O. Line segments AO, BO, CO and DO are represented by $3x + 2y$, $x + 18$, $7x - 6y$ and $5x - 2y$ respectively. Is $ABCD$ a rectangle? [Leave all work on your paper.] [10]

160

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–12) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 Diagonal AC divides parallelogram $ABCD$ into two triangles of which angle ACB and angle ... are corresponding parts. 1.....

2 A regular hexagon is inscribed in a circle. If the radius of the circle is 2 inches, the perimeter of the hexagon is ... inches. 2.....

3 If the diameter of a circle is 10, the area of the circle in terms of π is .... 3.....

4 The sides of a given triangle are 6, 9 and 12 inches. The perimeter of a similar triangle whose shortest side is 4 inches is ... inches. 4.....

5 If in right isosceles triangle ABC the bisector of the right angle C is 6 inches in length, then the hypotenuse AB is ... inches. 5.....

6 A diameter of a circle is parallel to a chord that subtends an arc of 100°. One of the arcs intercepted between the diameter and the chord contains ... degrees. 6.....

7 If the diagonals of a rhombus are 14 and 48, one side of the rhombus is .... 7.....

8 The area of a trapezoid is 55 square inches. If the bases are 14 inches and 8 inches, then the altitude is ... inches. 8.....

9 From a point outside a circle a tangent and a secant are drawn to the circle. If the secant is 27 inches and its external segment is 3 inches, the tangent is ... inches. 9.....

10 If two adjacent sides of a parallelogram are 6 inches and 8 inches and the included angle is 30°, the area of the parallelogram is ... square inches. 10.....

11 One of the arcs intercepted by two tangents drawn to a circle from an external point contains 260°. The number of degrees in the angle formed by the tangents is .... 11.....

12 The sides of a right triangle are 3, 4 and 5. The smallest angle of this triangle, correct to the *nearest degree*, contains ... degrees. 12.....

13 If two rectangles have equal perimeters, must they have equal areas? [Answer *yes* or *no*.]. 13.....

14 If the area of an equilateral triangle is $9\sqrt{3}$, what is the length of one side? 14.....

[OVER]

Directions (questions 15–19) — Indicate the correct answer to each question by writing on the dotted line at the right the letter *a*, *b* or *c*.

15 If two right triangles have an acute angle of one equal to an acute angle of the other, then the triangles must be (*a*) congruent, (*b*) similar or (*c*) equal. 15.....

16 If the diagonals of a rhombus are 8 and 12, the area of the rhombus is (*a*) 24, (*b*) 48 or (*c*) 96. 16.....

17 A good citizen votes at election time. Jones votes at election time. Therefore Jones is a good citizen.

The above reasoning (*a*) is unsound and is an example of indirect reasoning, (*b*) is sound and is an example of direct reasoning or (*c*) is unsound and is an example of reasoning from a converse. 17.....

18 The locus of the center of a circle having a given line segment as a chord is (*a*) a line parallel to the given line segment, (*b*) a circle having the given line segment as a diameter, or (*c*) the perpendicular bisector of the given line segment. 18.....

19 If the circumference of a circle is C, the radius of this circle is

(*a*) $\frac{C}{\pi}$, (*b*) $\frac{C}{2\pi}$ or (*c*) $\sqrt{\frac{C}{\pi}}$ 19.....

Directions (questions 20–23) — Indicate whether each statement is true or false by writing the word *true* or the word *false* on the dotted line at the right.

20 The altitudes of a triangle always intersect inside the triangle. 20.....

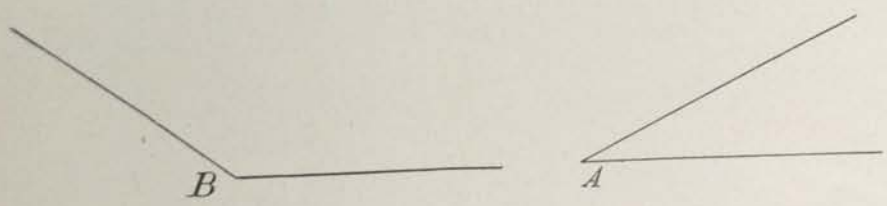
21 In triangle ABC, if AB is greater than AC, then angle C is greater than angle B. 21.....

22 If three sides of a triangle are unequal, the medians of the triangle will not pass through the same point. 22.....

23 If an exterior angle of a regular polygon contains 40°, the polygon has nine sides. 23.....

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Find by construction an angle which is equal to the difference between the given obtuse angle B and the given acute angle A.



25 Construct an isosceles triangle whose base is the given line segment b and whose altitude upon b is the given line segment h.

b

h