The University of the State of New York

278TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, June 18, 1940 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II, III and IV

Write at top of first page of answer paper to groups II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

Fill in the following lines:

Name of school..Name of pupil..

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–13) — Write on the dotted line at the right of each question the expression which, when inserted in the corresponding blank, will make the statement true.

1 A central angle of 40° intercepts an arc of ... degrees.   1................

2 The angle formed by two chords intersecting within a circle is measured by one half the ... of the intercepted arcs.   2................

3 The number π is a constant which represents the ratio between the circumference and the ... of a circle.   3................

4 If the area of a circle is 16π, the radius of the circle is ....   4................

5 The center of a circle circumscribed about a triangle is equidistant from the three ... of the triangle.   5................

6 If one of the equal angles of an isosceles triangle contains 65°, the smallest angle of the triangle contains ... degrees.   6................

7 Two triangles with equal bases and equal altitudes are always ....   7................

8 Two corresponding sides of two similar triangles are 8 and 12. If an altitude of the smaller triangle is 6, the corresponding altitude of the larger triangle is ....   8................

9 If a secant to a circle from an external point is 9 and its external segment is 4, the length of the tangent from that point is ....   9................

10 If two triangles are similar and the area of one is four times the area of the other, a side of the larger triangle is ... times the corresponding side of the smaller.   10................

11 If the hypotenuse of a right triangle is 17 inches and one leg is 15 inches, the other leg is ... inches.   11................

12 The area K of a regular polygon of n sides, whose apothem is a and whose side is s, is given by the formula $K = ...$ 12................

13 ABC is an isosceles triangle with AB and AC the equal sides. If angle B contains 35° and BC equals 20, the altitude upon BC, correct to the *nearest integer*, is ....   13................

Directions (questions 14–17) — Indicate the correct answer to each question by writing on the dotted line at the right the letter *a*, *b* or *c*.

14 If the altitude CD is drawn to the hypotenuse AB of the right triangle ABC, AC is the mean proportional between (*a*) AD and DB, (*b*) AB and AD or (*c*) AB and BC.   14................

15 AC and BD are diagonals of parallelogram $ABCD$. If $\angle DAB = 50°$, then (*a*) $AC = BD$, (*b*) $AC < BD$ or (*c*) $AC > BD$.   15................

16 If in right triangle ABC, $\angle A = 30°$ and $\angle B = 60°$, then (a) $AC = 2 BC$, (b) $AC = \frac{1}{2} AB$ or (c) $AC = BC \sqrt{3}$.

16..................

17 The diagonals of a rectangle are always (a) equal and perpendicular to each other, (b) perpendicular and bisect each other or (c) equal and bisect each other.

17..................

Directions (questions 18–22) — Indicate whether each statement is *always* true, *sometimes* true or *never* true by writing on the line at the right the word *always*, *sometimes* or *never*.

18 If two chords of a circle intersect, the product of the segments of one chord is equal to the product of the segments of the other.

18..................

19 The diagonals of a trapezoid are equal.

19..................

20 Two right triangles are congruent if the legs of one are equal to the legs of the other.

20..................

21 The sum of the interior angles of any quadrilateral is equal to the sum of its exterior angles.

21..................

22 If in a given circle arc AB equals arc BC, then chord AC is twice chord AB.

22..................

Directions (questions 23–25) — Leave all construction lines on the paper.

23 Given line segments a, b and c
    Construct line segment x such that
        $a : b = c : x$

a _____

b _____

c _____

24 Construct an angle of 30°.

r _____

25 Construct the locus of points equidistant from the two given parallel lines r and s.

s _____

[4]

Group II
Answer two questions from this group.

26 Prove that tangents drawn to a circle from an external point are equal. [10]

27 *ABCDE* is an equilateral pentagon inscribed in a circle. *FG* is tangent to the circle at point *A*. Prove that *FG* is parallel to *CD*. [10]

28 Prove that the area of a trapezoid is equal to one half the product of its altitude and the sum of its bases. [10]

Group III
Answer two questions from this group.

29 *RS* is a diameter of a circle and *NS* is a chord. At *D*, a point on *RS* extended through *S*, a line perpendicular to *RD* is drawn. *NS* extended meets this perpendicular at *M*. *RN* is drawn.
  a Prove that $\triangle RNS$ and $\triangle SDM$ are similar. [6]
  b If *RS* = 30, *MS* = 16 and *NS* = 10, find the length of *SD*. [4]

30 The longer diagonal of a rhombus is 24 feet and the shorter diagonal is 10 feet.
  a Find the perimeter of the rhombus. [4]
  b Find, correct to the *nearest degree*, the angle which the longer diagonal makes with a side of the rhombus. [6]

31 The radius of a circle is 12 and a minor segment of this circle has a chord equal to the radius.
  a Find the perimeter of the minor segment. [4]
  b Find the area of the minor segment. [6]
[Answers may be left in radical form and in terms of π.]

Group IV
Answer one question from this group.

32 *a* What is the locus of the vertex of the right angle of a right triangle whose hypotenuse is a given line segment? [1]
  b What is the locus of the vertex of angle *C* of triangle *ABC* in which *AB* and the median *m* upon *AB* are given line segments? [1]
  c What is the locus of the vertex of angle *E* of triangle *DEF* in which *DF* and the altitude *h* upon *DF* are given line segments? [2]
  d If the hypotenuse and the median upon the hypotenuse of a right triangle are given, is the triangle determined? Explain. [1, 2]
  e If three line segments, *s*, *m* and *h*, are chosen at random to represent a side of a triangle, the median and the altitude to that side respectively, is it always possible to construct the triangle? Explain. [1, 2]

33 *a* Explain what is meant by a converse of a proposition in geometry. [2]
  b Does every proposition have a converse? [1]
  c State a proposition of which a converse is not true and write this converse. [3]
  d State *two* converses of the following proposition: The diameter of a circle perpendicular to a chord of the circle bisects the chord and the arcs determined by the chord. [4]