The University of the State of New York

269TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, June 15, 1937 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–8) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 An angle formed by two chords intersecting within the circle is equal in degrees to one half the ... of the intercepted arcs. *Ans*.....

2 The bisectors of the angles of a triangle meet in a point which is equidistant from the three ... of the triangle. *Ans*.....

3 If the bases of a trapezoid are 12 inches and 18 inches, the length of the line segment joining the mid-points of the nonparallel sides is ... inches. *Ans*.....

4 The area K of a trapezoid in terms of its altitude h and its bases b and b' is given by the formula $K =$ .... *Ans*.....

5 The area K of an equilateral triangle whose side is a is given by the formula $K =$ .... *Ans*.....

6 If each exterior angle of a polygon is 40°, the number of sides of the polygon is .... *Ans*.....

7 If an acute angle of a parallelogram is 60°, an obtuse angle of the parallelogram is ... degrees. *Ans*.....

8 A tangent and a secant are drawn to a circle from a point outside the circle. If the tangent is 14 inches long and the secant is 28 inches long, the length of the external segment of the secant is ... inches. *Ans*.....

Directions (questions 9–13) — Indicate whether each of the following statements is *always* true, *sometimes* true or *never* true by writing the word *always, sometimes* or *never* on the dotted line at the right.

9 The area of a rhombus is equal to one half the product of its diagonals. *Ans*.....

10 If the angle between two tangents to a circle is 60°, the triangle formed by these tangents and the chord joining the points of contact is equiangular. *Ans*.....

11 A line passing through the mid-point of a chord of a circle passes through the center of the circle. *Ans*.....

12 If two angles of a triangle are 30° and 60°, the side opposite the 60° angle is twice the side opposite the 30° angle. *Ans*.....

13 If C is the mid-point of the arc AB of a circle, then the chord AB is twice the chord AC. *Ans*.....

[OVER]

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Directions (questions 14–17) — Indicate the correct answer to each of the following questions by writing the letter a, b, or c in the space at the right.

14 If two angles of one triangle are equal respectively to two angles of another triangle, the triangles must be (*a*) congruent, (*b*) similar or (*c*) equal. *Ans*.....

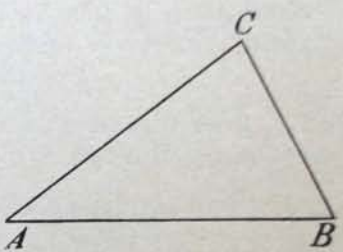
15 The number of points which are equidistant from two given parallel lines and, at the same time, are equidistant from two given points on one of these lines is (*a*) one, (*b*) two or (*c*) three. *Ans*.....

16 If the radius of a circle is increased by x, the circumference of the circle is increased by (*a*) x, (*b*) $2x$ or (*c*) $2\pi x$. *Ans*.....

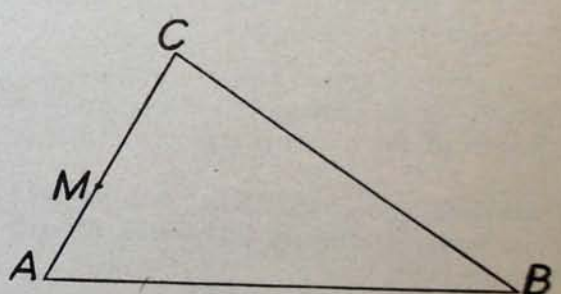
17 The areas of any two regular polygons which have equal perimeters are to each other as (*a*) their apothems, (*b*) the squares of their apothems or (*c*) their perimeters. *Ans*.....

Directions (questions 18–20) — Leave all construction lines on the paper.

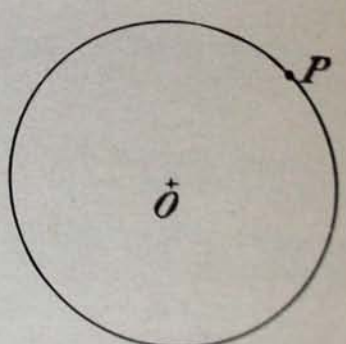
18 Find the center of the circle that can be circumscribed about triangle ABC.



19 Through the point M construct a line which will divide the sides AC and BC of triangle ABC proportionally.



20 Construct a tangent to the circle O at point P.



See instructions for groups II and III on page 1.

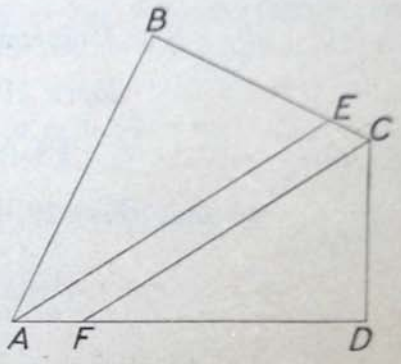
Group II

Answer three questions from this group.

21 Prove that if in the same circle or in equal circles two chords are equal, they are equidistant from the center. [10]

22 Prove that the areas of two similar triangles are to each other as the squares of any two corresponding sides. [10]

23 In the figure at the right, $ABCD$ is a quadrilateral, angles B and D are right angles, AE bisects angle A, and CF bisects angle C. Then AE is parallel to CF.



Below is a possible proof for this exercise.

Give a reason for *each* of the following statements:

1	$\angle DAB + \angle B + \angle BCD + \angle D = 360°$	[2]
2	$\angle B + \angle D = 180°$	[1]
3	$\angle DAB + \angle BCD = 180°$	[1]
4	$\angle BAE + \angle BCF = 90°$	[1]
5	$\angle BAE + \angle BEA = 90°$	[2]
6	$\angle BCF = \angle BEA$	[1]
7	$AE \parallel CF$	[2]

24 AB is a diameter of a circle and at B a tangent to the circle is drawn. From A, a line is drawn intersecting the circle at C and the tangent at D. Prove $AC : AB = AB : AD$ [10]

25 In the triangle ABC, D is any point in AC and E is any point in CB. DE is drawn. Prove that the sum of AC and CB is greater than the sum of AD, DE and EB. [10]

Group III

Answer two questions from this group.

26 The radius of a circular flower bed is 30 feet and this bed is surrounded by a path 3 feet wide. Find the cost of paving the path at 25 cents per square foot. (Use $\pi = \frac{22}{7}$) [10]

27 Given two lines m and n intersecting each other at right angles, and point P on one of these lines

a Describe completely the locus of points which are at a given distance s from P. [2]

b Describe completely the locus of points equidistant from m and n. [3]

c How many points are there which will satisfy both conditions given in *a* and *b* if P is 4 inches from the intersection of m and n and

(1) s is 5 inches long [2]

(2) s is 1 inch long [3]

28 The altitude of a triangle is 12 inches and it divides the vertex angle into two angles of 20° and 45°.

a Find the lengths of the segments of the base. [6]

b Find, correct to the *nearest square inch*, the area of the triangle. [4]