PLANE GEOMETRY

Wednesday, June 19, 1935 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

[1]

Fill in the following lines:

Name of school..Name of pupil..

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–17) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true. No work need be shown.

1 A segment of a circle is bounded by an arc and its .... *Ans............*

2 A median of a triangle divides it into two ... triangles. *Ans............*

3 In a certain 30°–60° right triangle, the hypotenuse is 20; then the shortest side of the triangle is .... *Ans............*

4 If the areas of two circles are in the ratio 9 : 4, and the radius of the larger circle is 6, the radius of the smaller circle is .... *Ans............*

5 If the sum of the interior angles of a polygon is 540°, the number of sides of the polygon is .... *Ans............*

6 If the area of a circle is 49 π, the circumference of the circle is .... [Answer may be left in terms of π.] *Ans............*

7 If the diagonals of a parallelogram are equal and bisect the angles through which they pass, the parallelogram is a .... *Ans............*

8 If the altitude of an equilateral triangle is 6, the radius of the circle circumscribed about the triangle is .... *Ans............*

9 The altitude CH is drawn to the hypotenuse AB of right triangle ABC; then angle HCB equals angle .... *Ans............*

10 If two circles intersect, the distance between their centers is ... the sum of the radii. [Answer *equal to, less than* or *greater than.*] *Ans............*

11 In triangle ABC, angle $C = 90°$, angle $A = 46°$, $AC = 60$; then the length of BC is .... *Ans............*

12 The diagonals of a rhombus are 30 inches and 16 inches; then the length of each side of the rhombus is ... inches. *Ans............*

13 A chord of a circle increases in length as its distance from the center .... *Ans............*

14 The locus of points outside circle O and at a distance of d inches from it is a .... *Ans............*

15 A rectangle and a triangle have the same base and the area of the triangle is twice that of the rectangle; then the altitude of the triangle is ... times the altitude of the rectangle. *Ans............*

16 AB is a diameter of a circle and AP is perpendicular to AB. PB cuts the circle at C and chord AC is drawn. Then AB is the mean proportional between PB and .... *Ans............*

17 The acute triangle ABC is inscribed in circle O and radius OR intersects side AC at right angles. If radius OA is drawn, angle AOR equals angle ... of triangle ABC. *Ans............*
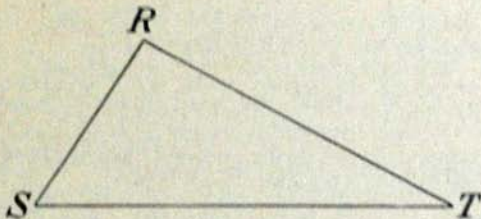
[3]

Directions (questions 18–20) — Leave all construction lines on the paper.

18 Find the center of the circle that can be inscribed in triangle ABC.
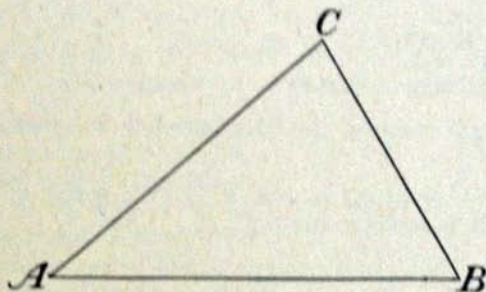
19 Construct a line parallel to ST through vertex R of triangle RST.

20 Construct the mean proportional between the given line segments a and b.

a

b

[4]

190

See instructions for groups II and III on page 1.

Group II
Answer three questions from this group.

21 Prove that two triangles are congruent if the three sides of one are equal respectively to the three sides of the other. [10]

22 Prove that the area of a regular polygon is equal to one half the product of its perimeter and its apothem. [10]

23 In triangle ABC, AC is greater than BC, AE bisects angle A and BF bisects angle B. If AE intersects BF in D, prove that AD is greater than BD. [10]

24 Prove that if two diagonals of an inscribed regular pentagon intersect, the longer segment of one diagonal is equal to the longer segment of the other diagonal. [10]

25 Two tangents are drawn to a circle at the ends of a diameter and a third tangent is intercepted between them. If the radius is drawn to the point of contact of the third tangent, prove that it is the mean proportional between the segments of that tangent.

Given: Circle O, diameter AB and tangents AD, BC and DTC; also radius OT.

To prove: $DT:TO = TO:TC$

Construction: Join O to D and C.
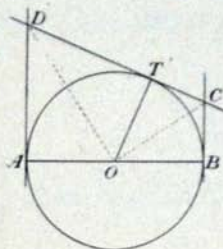
Below are given the statements for a proof of the above theorem but they are not arranged in logical order. *Do not copy the above material.* Rearrange the statements in logical order on your answer paper and assign a reason for *each* statement. [10]

∴ ∠ $DOC = 1$ rt. ∠

∴ $AD \parallel BC$

∴ ∠ $ODC + $ ∠ $OCD = 1$ rt. ∠

$AD \perp AB$; $BC \perp AB$

∴ ∠ $ADC + $ ∠ $BCD = 1$ st. ∠

Also, $TO \perp DC$

∴ $DT:TO = TO:TC$

But OD bisects ∠ ADC ; OC bisects ∠ BCD

Group III
Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Irrational results may be left in the form of π and radicals unless otherwise stated.

26 The base of a triangle is 32 inches and the corresponding altitude is 20 inches; find the area of the trapezoid cut off by a line parallel to the base and 15 inches from it. [10]
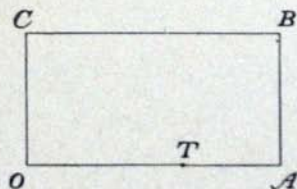
27 Two equal chords are produced until they meet outside the circle, forming an angle of 18°. The smaller arc intercepted by the sides of this angle contains 24°. Find each angle of the quadrilateral formed by joining the ends of the chords. [10]

28 Tangents are drawn from an external point to a circle whose radius is 12 inches. If the minor arc intercepted by these tangents is $\frac{1}{5}$ of the circle, find, correct to the *nearest inch*, the distance from the point to the center of the circle. [Use numerical trigonometry.] [10]

29 $OABC$ is a rectangular field on one side of which a tree is located at a point T, 42 yards from the corner O. A treasure is buried in this field 26 yards from the tree and 32 yards from side OC.

  a Explain how you would locate the point where the treasure is buried. [6]

  b How far is this point from side OA? [4]

[2]