The University of the State of New York

242b High School Examination

PLANE GEOMETRY

Wednesday, June 20, 1928 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry.

The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in plane geometry.

Answer eight questions, including not more than two from each of groups I and II, and at least two from each of groups III and IV.

Group I

Do not answer more than two questions from this group.

1 Prove that if in the same circle, or in equal circles, two chords are equal, they are equidistant from the center. [12½]

2 Prove that the areas of two similar triangles are to each other as the squares of any two corresponding sides. [12½]

3 *a* State the theorem about congruent triangles used in proving that **two triangles are congruent if the three sides of one are equal respectively to the three sides of the other.** [2½]

b From the definition of similar triangles what remains to be shown in proving that **if two triangles have the three angles of one equal respectively to the three angles of the other, the triangles are similar?** [2½]

c State the theorem about areas used in proving that **the area of a regular polygon is equal to one half the product of its perimeter and its apothem.** [2½]

d State the fundamental principle about measurement of angles used in proving that **an angle inscribed in a circle is measured by one half its intercepted arc.** [2½]

e State *two* things that must be shown in proving that **the locus of points equidistant from two given points is the perpendicular bisector of the line joining them.** [2½]

Group II

Do not answer more than two questions from this group.

Drawings in this group should be constructed accurately with ruler and compasses. Leave all construction lines on the paper.

4 Given circle O and point P outside the circle; through P construct a line that shall be tangent to circle O. [12½]

5 Given a triangle ABC

a Construct the circle that circumscribes this triangle. [9½]

b State where the center of the circle would be with reference to the triangle when the triangle is (1) acute, (2) right, (3) obtuse. [3]

6 Given a circle O and an inscribed angle ABC

a Find in arc AC the point P which is equidistant from lines BA and BC. [8]

b Construct in the drawing a line that represents this distance. [4½]

Group III

Answer at least two questions from this group.

7 ABC is an isosceles triangle in which angle A equals angle B. The bisectors of angle A and angle B meet BC and AC at D and E respectively.

a Prove that triangles ABD and ABE are congruent. [7]

b Prove that triangles ADC and BEC are congruent. [5½]

8 P is any point within a circle whose center is O. Line OP, extended through P, cuts the circle at point R. Prove that PR is shorter than any other line from P to the circle. [12½]

9 AB is a chord of a given circle. C is the mid-point of one intercepted arc and chord CD cuts chord AB in point E. If chord AC is drawn, prove that AC is the mean proportional between CE and CD. [12½]

10 $ABCD$ is a quadrilateral and E, F, K and L are the mid-points of the sides, taken in order.

a Prove that $EFKL$ is a parallelogram. [8]

b Prove that the perimeter of the parallelogram equals the sum of the diagonals of the quadrilateral. [4½]

[OVER]

Group IV

Answer at least two questions from this group.

Irrational results may be left in the form of π and radicals unless otherwise stated.

11 The area of any triangle is expressed by the formula $A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$, where a, b and c are the lengths of the sides and s is one half their sum.

a Using the above formula, find the area of the triangle whose sides are 17, 25 and 26. [9]

b Find the length of the altitude on side 17. [$3\frac{1}{2}$]

12 The radius of a circle is 8; find to the *nearest tenth* the area of the minor segment of the circle formed by one side of an inscribed equilateral triangle. [Assume $\pi = 3.14$] [$12\frac{1}{2}$]

13 The diagonals of a rhombus are in the ratio 3:4 and the area of the rhombus is 96 square inches; find one side of the rhombus. [$12\frac{1}{2}$]

14 AB is a chimney built on level ground, B being the base of the chimney. At a point C on the same level as B, the angle BCA is found to be 60°. At point D 50 feet from C on line BC extended, the angle BDA is found to be 30°. Find to the *nearest foot* the height of the chimney. [$12\frac{1}{2}$]