The University of the State of New York

219TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Thursday, June 20, 1918—1.15 to 4.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry. The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in plane geometry.

Answer eight questions.

1 Prove that if two triangles have the three sides of one respectively equal to the three sides of the other, the triangles are congruent.

2 Prove that the square on (or of) the hypotenuse of a right triangle is equivalent to the sum of the squares on (or of) the other two sides.

3 Prove that if through a point outside a circle a tangent and a secant are drawn, the length of the tangent is the mean proportional between the whole secant and its external segment.

4 Prove that the area of a circle is equal to half the product of its circumference and its radius; and also to the product of the square of the radius and the constant number π.

5 The area of a square inscribed in a circle is 16 square inches. Find the area of the circle.

6 The radius of a circle is 15 inches. Through a point 5 inches from the center a chord is drawn. What is the product of the two segments of the chord? What is the length of the shortest chord that can be drawn through that point?

7 The base of a triangle is 20 feet; the other sides are 16 feet and 10 feet. A line parallel to the base cuts off 2 feet from the lower end of the shorter side. Find the segments of the other side and the length of the parallel.

8 Construct a line tangent to a given circle and parallel to a given line outside the circle. [To receive credit construction lines must be shown.]

9 *a* An exterior angle of a regular polygon equals one fifth of a right angle. Find the number of sides of the polygon.

b The difference between two angles inscribed in the same circle is 20°. What is the difference between the two central angles subtended by the arcs of the inscribed angles?

c If the side of one equilateral triangle is equal to the altitude of another, what is the ratio of their areas?

d Two sides of a triangle are 3′ and 8′. What are the numerical limits of the third side?

10 ABC is a triangle. D is the foot of the perpendicular from A on BC, P is the middle point of BC, X is a point on BC such that $XP=PD$. If the line through P perpendicular to BC meets AX in M, prove that

$MB=MC$

$MX=MA$

11 From an external point P, a secant PM is drawn to a circle so that it is bisected by the circle at N. MD is the diameter through M. Prove that PD is equal to the diameter and state a simple method for drawing a secant from an external point to a circle so that it will be bisected by the circle.

12 Prove that a line perpendicular to a side of a right triangle at its middle point passes through the mid point of the hypotenuse.

13 ABC is a triangle and AD, BE and CF are its medians. DH is drawn equal and parallel to BE, and cuts AC. Prove that HA is equal and parallel to CF.