

PLANE GEOMETRY

Thursday, June 22, 1916—1.15 to 4.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry. The minimum time requirement is five recitations a week for a school year. Name the author of the textbook you have used in plane geometry.

Answer eight questions, including the eleventh.

Assign 16 credits to the eleventh question and 12 credits to each of the others.

1 Prove that in an isosceles triangle the angles opposite the equal sides are equal.

2 Prove that an angle formed by two chords intersecting each other within a circle is measured by one half the sum of the arc intercepted between its sides and the arc intercepted between the sides of its vertical angle.

3 Prove that if the angles of two triangles are respectively equal to each other the triangles are similar.

4 Prove that the areas of two circles are to each other as the squares of the radii.

5 The minute hand and the hour hand of a watch are  $\frac{3}{4}$  of an inch and  $\frac{1}{2}$  of an inch long respectively; how many inches farther does the extremity of the minute hand travel in one day than the extremity of the hour hand?

6 a How would you draw the shortest possible chord through a given point within a circle?

b How would you draw the shortest line to a given circle from a given point within the circle?

c How many points are required to determine a straight line? an angle? a circle?

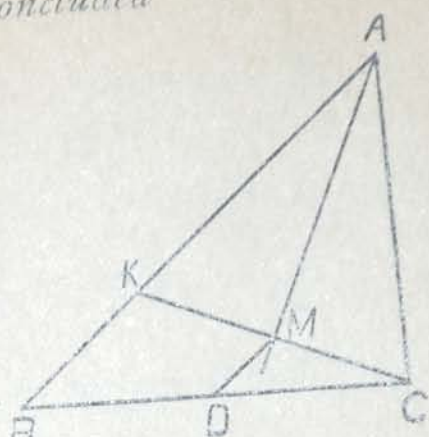
d If the radius of a circle is multiplied by 3, by what number is the circumference multiplied? the area?

7  $AB$  is the diameter of a circle,  $P$  any point on the circumference.  $AP$  is produced to  $L$  so that  $AP$  equals  $PL$ . Prove that  $BL$  is equal to the diameter of the circle.

8  $ABCD$  is a quadrilateral having  $AB$  equal to  $AD$  equal to two inches. Angle  $A$  equals  $90^\circ$ , angle  $B$  equals  $120^\circ$ , angle  $D$  equals  $60^\circ$ . Construct the figure, showing all construction lines.

## PLANE GEOMETRY—concluded

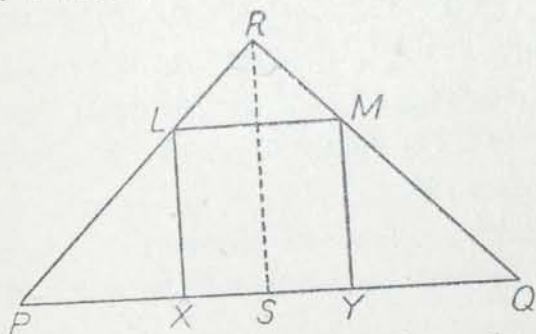
9  $AB$  equals 7,  $BC$  equals 5.5,  $AC$  equals 4.6.  $AM$  bisects  $\angle BAC$ , and  $CM$  is the perpendicular from  $C$  to  $AM$ .  $CM$  is produced to meet  $AB$  at  $K$ , and  $D$  is the mid point of  $BC$ . Find the length of  $BK$  and of  $DM$ .



10 For each of the following theorems (no proof required) draw the figure and state what is given and what is to be proved in terms of letters on the figure:

- a If two circles are externally tangent to each other, any two straight lines drawn through the point of contact will be cut proportionally by the circles.
- b If through the extremities of each diagonal of a quadrilateral lines parallel to the other diagonal are drawn, a parallelogram double the given quadrilateral will be formed.

11  $PQR$  is a right isosceles triangle, right-angled at  $R$ .  $L$  and  $M$  are the trisection points nearest  $R$  of the sides  $RP$  and  $RQ$ .  $X$  and  $Y$  are the trisection points of the hypotenuse.



Draw  $RS$  to  $S$ , the mid point of  $PQ$ , and note that

$$\frac{PX}{PS} = \frac{QY}{QS} = \frac{LX}{RS} = \frac{MY}{RS}$$

Assign a reason to each of the following eight questions:

- 1 Why does  $LX = MY$ ?
- 2 Why does  $\frac{PX}{XS} = \frac{PL}{LR} = 2$ ?
- 3 Why is  $LX$  parallel to  $RS$ ? Why is  $MY$  parallel to  $RS$ ?
- 4 Why is  $LX$  parallel to  $MY$ ?
- 5 Why is  $LMYX$  a parallelogram?
- 6 Why is  $\angle QSR$  a right angle?
- 7 Why is  $\angle QYM$  a right angle?
- 8 Why is  $LMYX$  a square?