University of the State of New York

211TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, June 16, 1914—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry. The minimum time requirement is five recitations a week for a school year. Name the author of the textbook you have used in your study of plane geometry.

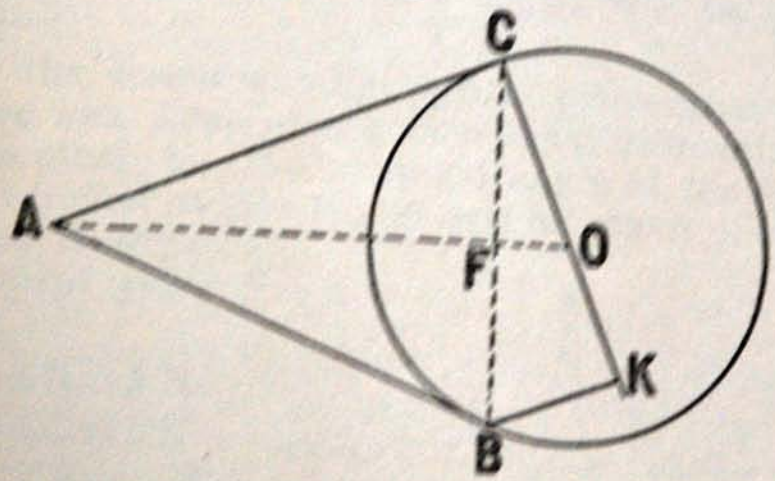
Answer the first question and seven of the others.

Assign 16 credits to the first question and 12 credits to each of the others.

1 Assign a reason to *each* of the eight steps given in the following proof:

THEOREM

From a point A tangents AB and AC are drawn to a circle whose center is O, and BK is drawn perpendicular to CO produced. Prove that AC : CK = CO : BK



Proof Draw BC and OA intersecting at F.

1 $\angle BAF = \angle FAC$
2 $\angle FBA = \angle ACF$
3 $\angle AFB = \angle AFC$
4 $AO \perp BC$
5 $AC \perp CK$
6 $CO \perp BK$
7 $\triangle ACO$ is similar to $\triangle CKB$
8 $AC : CK = CO : BK$

2 Prove that if a point is equidistant from the sides of an angle it is in the bisector of the angle.

3 Prove that the area of a regular polygon is equal to one half the product of its apothem and perimeter. [10]

State (without proof) the corresponding theorem for the circle. [2]

4 Prove that two similar polygons may be divided into the same number of triangles similar each to each and similarly placed.

5 Wishing to find the distance between A and B on opposite sides of a river, a man at A lays off AD perpendicular to AB. He locates E at the mid point of AD. He then lays off DF perpendicular to AD, choosing F so that F, E and B are in the same straight line. Draw a figure and state what one line he would measure in order to know the length of AB. [3]

Prove that your statement is correct. [4]

6 ABCDE is a regular pentagon inscribed in a circle.

a Prove that the $\angle A$ is trisected by AC and AD.

b At what angle do AD and CE intersect?

c Prove that the tangent to the circle at A is parallel to CD.

7 It is desired to locate 8 trees equidistant from one another, each 20 feet from a given point. *Construct* the figure, showing all construction lines. [No credit will be granted for explanation.]

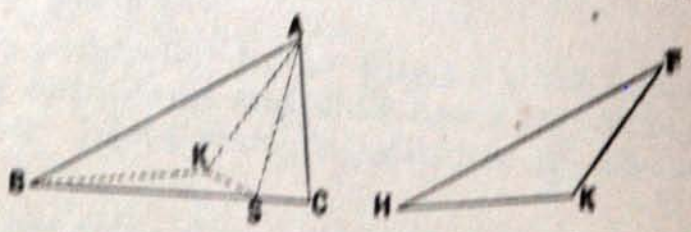
8 From the accompanying figure, prove that if two triangles have two sides of the one equal respectively to two sides of the other, but the included angle of the first greater than the included angle of the second, then the third side of the first is greater than the third side of the second.

Given $AB \cong FH$

$AC \cong FK$

$\angle A > \angle F$

AS bisects $\angle KAC$



[Credit will not be granted if this figure is not used.]

9 A straight line is drawn cutting two equal circles and parallel to their line of centers. Prove that the chords intercepted in the two circles are equal.

10 From a rectangular piece of cardboard $12'' \times 14''$, an isosceles trapezoid with bases $14''$ and $8''$ and sides $5''$, and a square with side $8''$ are cut; what fractional part of the cardboard is wasted? [11]

Draw to scale a figure illustrating this problem. [1]

11 What is the area of a circle whose arc of $22\frac{1}{2}°$ is $\frac{5\pi}{2}$ ft long?