High School Department

169TH EXAMINATION

PLANE GEOMETRY

Wednesday, June 19, 1901 — 9.15 a. m. to 12.15 p. m., only

Answer eight questions but **no more,** *including at least one from each of the three divisions. If more than eight are answered only the first eight answers will be considered. Draw carefully and neatly each figure in construction or proof, using letters instead of numerals. Arrange work logically. Each complete answer will receive* $12\frac{1}{2}$ *credits. Papers entitled to 75 or more credits will be accepted.*

First division 1 Define *five* of the following: adjacent angles, trapezoid, heptagon, perimeter, quadrant, segment, diagonal.

2 Prove that if two triangles have two angles and the included side of one respectively equal to two angles and the included side of the other, the triangles are equal in all respects.

3 Prove that an inscribed angle is measured by one half the arc intercepted by its sides. Give *three* cases.

4 Give *three* conclusions to the following and demonstrate *one* of them: if in a right triangle a perpendicular is dropped from the vertex of the right angle to the hypotenuse . . .

5 Prove that the area of a circle is equal to *a*) one half the product of the radius and the circumference, *b*) π times the square of the radius [Derive *b* from *a*].

Second division 6 Find the perimeter of an equilateral triangle that is equal in area to a semicircle whose radius is r.

7 The base of a polygon is 4 feet; find the base of a similar polygon whose area is $2\frac{1}{2}$ times the area of the given polygon.

8 The radius of a circle is 10 inches; find the area of the segment of the circle subtended by a chord equal to the radius.

9 Find the side of the regular octagon inscribed in a circle whose radius is 8 inches.

10 From a point without a circle are drawn a secant 25 inches long terminating in the concave arc and a tangent 15 inches long; find the external segment of the secant.

Third division 11 Prove that if three alternate sides of a regular hexagon are produced till they meet, an equilateral triangle is formed.

12 Prove that the bisectors of the interior angles of a parallelogram form a rectangle.

13 Show how to construct an isosceles triangle when the base and the vertical angle are given. Give proof.

14 Prove that the lines joining the middle points of the sides of any quadrilateral form a parallelogram whose perimeter is equal to the sum of the diagonals of the quadrilateral.

15 Find the loci of the centers of circumferences situated as follows: *a*) tangent to two given straight lines, *b*) passing through two given points, *c*) tangent to a given straight line and having a given radius.