The University of the State of New York

323D HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Monday, January 24, 1955 — 9.15 a.m. to 12.15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from this part.

26 Prove: If the opposite sides of a quadrilateral are equal, the quadrilateral is a parallelogram. [10]

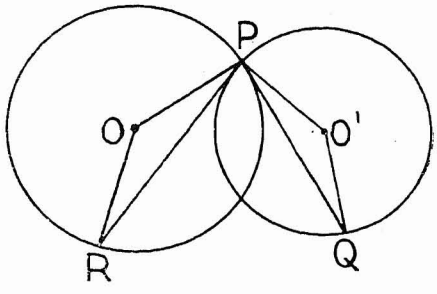
27 *a* Prove: An angle inscribed in a circle is measured by one half its intercepted arc. [Prove only the case in which one side of the angle is a diameter of the circle.] [8]

b Tell which of the following axioms is used to prove this theorem for the case in which the center of the circle lies outside the angle: [2]

(1) If equals are added to equals, the sums are equal.

(2) If equals are subtracted from equals, the remainders are equal.

28 In the accompanying figure unequal circles O and O' intersect at P. Chord PR is tangent to circle O' and chord PQ is tangent to circle O. Radii OP, OR, $O'P$, and $O'Q$ are drawn.



a Prove that

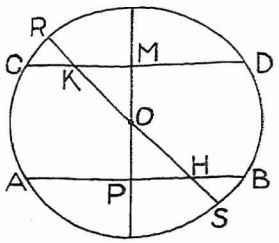
(1) angle OPR = angle $O'PQ$ [5]

(2) triangles OPR and $O'PQ$ are similar [3]

b If the ratio of chord PR to chord PQ is $a:b$, find the ratio of the area of circle O to the area of circle O'. [2]

[OVER]

29 Two chords AB and CD of a circle whose center is O are equal and parallel. A line through O perpendicular to AB intersects AB at P and CD at M. Diameter RS of the circle intersects AB at H and CD at K. Prove that:



a triangles HOP and KOM are congruent [6]

b $HS = KR$ [4]

30 Perpendicular lines r and s intersect at O, and P is a point on r.

a State *in full* the locus of points which are

(1) at a given distance k from P [3]

(2) equidistant from r and s [3]

b Find the number of points satisfying *both* conditions given in *a* if

(1) $OP = 6$ and $k = 8$ [1]

(2) $OP = 6$ and $k = 3\sqrt{2}$ [2]

(3) $OP = 6$ and $k = 3$ [1]

Part III

Answer two questions from this part. Show all work.

31 *a* The shorter base of an isosceles trapezoid is 17, its altitude is 8 and one of its angles is 45°. Find the area of the trapezoid. [4]

b Base AD of parallelogram $ABCD$ is represented by x. If angle A is 30°, and if side AB is 10, express the area of the parallelogram is terms of x. [4]

c If the trapezoid and the parallelogram are equal in area, find x. [2]

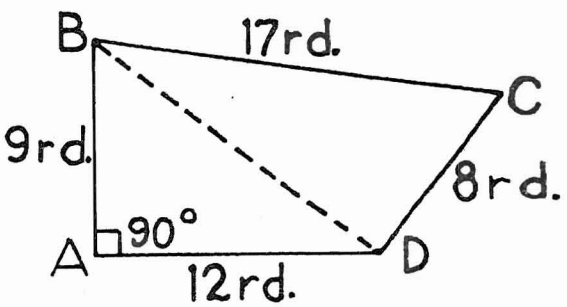
32 In rhombus $ABCD$ diagonal AC is drawn. If $AC = 20$ and angle $BAC = 27°$, find

a diagonal BD to the *nearest tenth* [4]

b the area of the rhombus to the *nearest integer* [2]

c side AB to the *nearest integer* [4]

33 A plot of land has the form of a quadrilateral whose sides are 9 rods, 17 rods, 8 rods, and 12 rods as indicated on the accompanying figure. Angle BAD equals 90°.



a Find the length of diagonal BD. [2]

b Show that angle BDC is a right angle. [2]

c Find to the *nearest tenth* of an acre the area of the field. [1 acre = 160 sq. rd.] [6]

34 AB is a diameter of a circle whose center is O. OC is a radius perpendicular to AB. K is a point on radius OB. Line CK is drawn and extended to meet the circle at D. At D a tangent is drawn to the circle and meets AB extended at P.

a Draw and letter a figure which represents the conditions given above. [2]

b If the number of degrees in arc DB is represented by $2x$, show that angle CDP and angle DKB are equal. [4]

c If $AP = 18$ and $KP = 12$, find the radius of the circle. [4]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

Directions (1–4): Write on the line at the right of *each* question the expression that, when inserted in the blank, will make the statement true.

1 If two parallel lines are cut by a transversal, the two interior angles on the same side of the transversal are .... 1.....

2 If line a is perpendicular to line b and line b is parallel to line c, then a is ... to c. 2.....

3 The angle formed by two secants intersecting outside the circle is measured by one half the ... of the intercepted arcs. 3.....

4 The areas of two parallelograms which have equal bases are to each other as their .... 4.....

Directions (5–12): Write on the line at the right the correct answer to *each* numerical exercise.

5 The line segment joining the midpoints of two sides of an equilateral triangle is 5. Find the *perimeter* of the triangle. 5.....

6 Two polygons are similar and the area of one is 9 times the area of the other. If a side of the smaller polygon is 5, find the corresponding side of the larger polygon. 6.....

7 AB is the hypotenuse of right triangle ABC and CD is the altitude on the hypotenuse. If $AD = 2$ and $CD = 8$, find DB. 7.....

8 Two chords, AB and CD, of a circle intersect at P. If $AP = 2$, $PB = 16$, and $CP = 4$, find PD. 8.....

9 A line parallel to base AB of triangle CAB intersects side AC at H and side BC at K. If $AH = 15$, $HC = 6$, and $BK = 10$, find KC. 9.....

10 Find the length of an arc whose central angle is 10° in a circle whose radius is 9 inches. [Answer may be left in terms of π.] 10.....

11 Find the area of a circle whose radius is 7. [Answer may be left in terms of π.] 11.....

12 In triangle ABC, angle $C = 90°$, side $AB = 100$, and side $BC = 89$. Find angle A to the *nearest degree*. 12.....

Directions (13–16): For *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, or *never* true, write the word *false*.

13 If the diagonals of a quadrilateral are equal, the quadrilateral is a rectangle. 13.....

14 Two right triangles are similar if the legs of one are proportional to the legs of the other. 14.....

15 A diameter which bisects one of two parallel chords, neither of which is a diameter, bisects the other also. 15.....

16 If two circles are externally tangent to each other, the greatest number of common tangents that can be drawn to both circles is four. 16.....

Directions (17–21): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

17 The center of a circle inscribed in a triangle is the point of intersection of (*a*) the bisectors of its angles (*b*) its altitudes (*c*) the perpendicular bisectors of its sides 17.....

18 If each interior angle of a regular polygon is 140°, the number of sides of the polygon is (*a*) seven (*b*) eight (*c*) nine 18.....

19 A median of a triangle divides the triangle into two triangles which are always (*a*) similar (*b*) equal in area (*c*) congruent 19.....

20 If in triangle ABC, angle $A = 50°$ and angle $B = 64°$, the longest side of the triangle is (*a*) AB (*b*) AC (*c*) BC 20.....

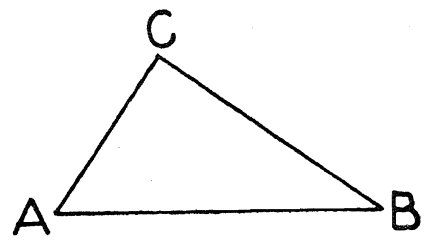
21 The area of an equilateral triangle whose side is 8 is (*a*) $16\sqrt{3}$ (*b*) $4\sqrt{3}$ (*c*) $16\sqrt{2}$ 21.....

22 Is statement *b* the converse of statement *a*? [Answer *yes* or *no*.]
a If the diagonals of a trapezoid are equal, the trapezoid is isosceles.
b The diagonals of an isosceles trapezoid are equal. 22.....

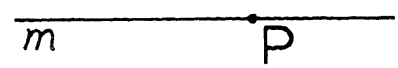
23 If a rhombus is *defined* as a parallelogram in which two adjacent sides are equal, does it follow from this definition that a square is a rhombus? [Answer *yes* or *no*.] 23.....

Directions (24–25): Leave all construction lines on your paper.

24 Divide side AB of triangle ABC into two parts which are in the ratio $AC : CB$



25 Given point P on line m. Construct the locus of the centers of circles tangent to line m at point P.



FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
PLANE GEOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 17–21, allow credit if the pupil has written the correct answer instead of the letter *a, b* or *c*.

(1) supplementary
(2) perpendicular
(3) difference
(4) altitudes
(5) 30
(6) 15
(7) 32
(8) 8
(9) 4
(10) $\frac{\pi}{2}$ inches
(11) 49π
(12) 63 *or* 63°
(13) false
(14) true
(15) true
(16) false
(17) *a*
(18) *c*
(19) *b*
(20) *a*
(21) *a*
(22) yes
(23) yes